Ph.D. DISSERTATION

Performance Analysis of Relay Selection Based on CDFs of SNRs in Wireless Relay Networks

무선 중계 네트워크에서 신호대잡음비의 누적분포함수 기반 중계기 선택 기법의 성능 분석

BY

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Abstract

Wireless relay technology is one of the most promising technologies for the future communication systems which provide coverage extension and better quality of service (QoS) such as higher data rate and lower outage probability with few excessive network loads. Due to its advantages, it has been adopted in wireless standards such as IEEE 802.16j and 3GPP LTE-Advanced.

In practice, since statistics of the channel between any two nodes vary depending on their locations, they are not identical which means that channels can experience different fading. When statistics of the channel are not identical, relay selection, which is one of the most useful techniques for wireless relay technology, can cause fairness problem that particular relays are selected more frequently than other relays. Especially, this problem can cause reduction of lifetime in the network with multiple relays having limited battery power. In this network, it is needed to focus on selection fairness for relays as well as reliability at end-users.

In this dissertation, to focus on both selection fairness for relays and reliability at end-users, we propose novel relay selection schemes based on cumulative distribution functions (CDFs) of signal-to-noise ratios (SNRs) in wireless relay networks. The

dissertation consists of two main results.

First, we propose the proactive and the reactive relay selection schemes based on CDFs of SNRs for one-way relay networks over Nakagami-m fading channels. If a relay is selected before the source transmission, it is called as proactive relay selection. Otherwise, if a relay is selected after the source transmission, it is called as reactive relay selection. For both the proactive and the reactive relay selection schemes, we analyze average relay fairness by deriving relay selection probability. For the proactive relay selection scheme, we obtain diversity order by deriving the integral and asymptotic expressions for outage probability. Also, for the reactive relay selection scheme, we obtain diversity order by deriving the exact closed-form and asymptotic expressions for outage probability. Numerical results show that the analytical results of the proposed schemes match the simulation results well. It is shown that the proposed schemes guarantee strict fairness among relays and extend network lifetime. Also, it is shown that diversity order depends on the number of relays and fading severity parameters.

Second, we propose the proactive and the reactive relay selection schemes based on CDFs of SNRs for two-way relay networks over Nakagami-m fading channels. For the proactive relay selection scheme, we analyze average relay fairness by deriving relay selection probability. Also, we analyze diversity order by deriving the integral and asymptotic expressions for outage probability. For the reactive relay selection scheme, we analyze average relay fairness by deriving the integral and asymptotic expressions for relay selection probability. Also, we obtain diversity order by deriving

the asymptotic expression for outage probability. Numerical results show that the analytical results of the proposed schemes match the simulation results well. It is shown that the proposed schemes guarantee strict fairness among relays and extend network lifetime. Also, it is shown that diversity order depends on the number of relays and fading severity parameters.

Keywords: Wireless relay technology, cumulative distribution function, relay selection, one-way relaying, two-way relaying, relay selection probability, average relay fairness, network lifetime, outage probability, diversity order.

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Chapter 1

Introduction

As the demands for wireless services are growing rapidly, the next-generation wireless systems are required to support enhanced the quality of service (QoS) such as data rate and reliability. Due to wireless channel impairment and lack of wireless resources, it needs to enhance the QoS with few excessive loads such as the complexity, cost, and power consumption. As one of the attractive approach, wireless relay technology has been studied widely. In the wireless relay technology, the transmission between the source and destination becomes more reliable with the help of relays with a few excessive network loads. Due to advantages of relay usage, the wireless relay technology has been attracting a lot of interest from both academia and industry. Also, the applications to the wireless systems have been studied widely.

In this chapter, Section 1.1 provides the background of the wireless relay technology. Section 1.2 describes the outline of this dissertation. In Section 1.3, we provide the notations, the list of the abbreviations, and some mathematical definitions and functions used throughout the dissertation.

1.1 Background and Related Work

1.1.1 Diversity

Depending on which domain for diversity is utilized, diversity can be classified into three types: Time diversity, frequency diversity, and spatial diversity [1]-[3].

Time diversity can be achieved by transmission of identical information at different time slots, that is, reception of independent signals at different time slots. Similarly, frequency diversity can be achieved by using different frequencies to transmit identical information. Spatial diversity is achieved by using multiple antennas or multiple relays to transmit identical information. The multiple antennas or relays should be physically separated by proper distance to experience independent fading.

Depending on whether multiple antennas are utilized for transmission or reception, spatial diversity can be classified into two types: Receive diversity and transmit diversity [1], [4]. Receive diversity can be achieved by combining independent received signals at the receiver. There are many combining methods such as selective combining, maximal ratio combining, equal gain combining, and switched combining. In downlink networks, the major problem of using the receive diversity approach is the cost, size, and power of the remote units [1]. Hence, spatial diversity techniques have been applied to base stations to improve their reception quality. It is more economical to add equipment to base stations rather than the remote units [1]. Transmit diversity can be achieved by repeated transmission of identical information at the transmitter or by using multiple relays [4], [5]. Compared to receive diversity, transmit diversity is more difficult to implement due to the need of more signal processing at both the transmitter and the receiver.

1.1.2 Wireless Relay Technology

Wireless relay technology is one of most promising technologies for next-generation wireless systems which provide higher data rate and better quality of service (QoS) [6]-[18]. In relay communications, a source transmits its signal to a destination with the help of one or multiple intermediate relays. As the distance between two adjacent nodes decreases, the effect of wireless channel impairments such as path-loss is reduced. Relaying enables to provide coverage extension and enhanced capacity.

The basic concept of wireless relay technology was firstly introduced by Van der Meulen in 1971 by analyzing the upper and lower bounds on the capacity for a three-node network which consists of a single source, a single relay, and a single destination [20]. In 1979, Cover and El Gamal analyzed the capacity of degraded, reversely degraded, and feedback relay channels [21]. After these early works, the wireless relay technology did not have much attention for a long time due to difficulty of practical implementation. However, it has been changed since the concepts of information theory were successfully implemented. In 1998, Sendonaris *et al.* introduced the concepts of two-user cooperation in the framework of a code division multiple access system, where each of the two users is responsible for transmitting not only their own signal but also the signal of other user [22]-[24]. In 2000, Schein *et al.* investigated a real, discrete-time Gaussian parallel network which consists of a single source, two relays, and a single destination [25]. In 2002, Gastpar *et al.* analyzed the asymptotic capacity of a wireless network as the number of relays increases [26]. In 2003, Laneman *et al.* developed and analyzed space-time coded cooperative diversity protocols where the relays that can fully decode the received signal from a source at the first time slot use a space-time code to cooperatively relay to a destination [27]. Also, in 2004, they introduced various cooperation protocols such as amplify-and-forward (AF), decode-and-forward (DF), selective relaying, and incremental relaying [28]. All there works assume an unidirectional transmission through relays, which is referred to as one-way relaying.

Two-way relaying has been attracting much interest recently due to its advantage of spectral efficiency enhancement by using either superposition coding or physical-layer network coding at relays, compared to one-way relaying [29]-[39]. Rankov *et al.* introduced and analyzed various two-way relaying protocols where multiple nodes communicate with multiple partners via multiple AF or DF relays [29]-[31]. In 2007, Popovski *et al.* investigated the conditions for sum-rate maximization of two-way relaying [32]. In 2008, Oechtering *et al.* investigated the broadcast capacity region of two-phase two-way relaying in terms of the maximal probability of error [33]. Kim *et al.* derived performance bounds for bidirectional coded cooperation protocols for each of three DF protocols [34]. In 2009, Koike-Akino *et al.* developed various modulation schemes to optimize two-way relay networks where network coding is used at physical layer [35]. In 2010, Wilson *et al.* investigated joint physical layer and network layer coding for two-way relay networks and analyzed upper bounds on the exchange capacity [36]. In 2011, Vaze *et al.* investigated the problem of finding relay techniques that maximize the achievable rate region and achieve the optimal diversity-multiplexing tradeoff (DMT) in a two-way relay networks [37]. In 2012, Ong *et al.* derived the achievable equal-rate region for multi-user AWGN multi-way relay channel [38]. In 2013, Choi *et al.* analyze the performance of a two-way relay network with co-channel interference from multiple interferers [39]. All these works assume a dual-hop transmission.

Multi-hop relaying has been extensively investigated in both academia and industry in order to combat performance degradation caused by path loss [40]-[48]. In multi-hop relaying, when the direct path between a source and a destination is in deep fade, the source communicates with the destination via multiple intermediate relays. In 2003, Hasna *et al.* analyzed the end-to-end outage probability for multi-hop relay networks with AF relays over Nakagami-m fading channels [40]. In 2004, Boyer *et al.* investigated four different multi-hop protocols: Amplified relaying multi-hop protocol, decoded relaying multi-hop protocol, amplified relaying multi-hop diversity protocol, and decoded relaying multi-hop diversity protocol [41]. In 2006 and 2008, Hossain *et al.* investigated the multi-hop relay networks based on the automatic repeat request (ARQ) [42], [43]. In 2009, Gui *et al.* investigated routing strategies for wireless multi-hop networks to achieve full diversity order [44]. Yi *et al.* proposed an optimum relay ordering algorithm for the multi-branch multi-hop cooperative diversity networks [45]. In 2010, Vajapeyam *et al.* investigated distributed space-time coded protocols for wireless multi-hop networks [46]. In 2012, Jang *et al.* investigated a DF-based multi-hop transmission system where relays forward the source data simultaneously and derive the closed-form expression for outage probability [47]. In 2014, Wang *et al.* investigated generalized network coding schemes and analyzed closed-form expressions for the upper bound of the outage probability for multi-hop two-way relay networks [48]. All these works assume that a relay does not transmit and receive a signal simultaneously, which is referred to as the half-duplex relaying.

Full duplex relaying, where a relay can transmit and receive a signal simultaneously on the same channel, achieves up to twice the capacity than half-duplex relaying [49]-[55]. It has been attracting much interest due to advances on interference cancellation and antenna isolation to mitigate loop interference [56]-[60]. In 2006, Liu *et al.* investigated a communication protocol for full-duplex relay and theoretically analyzed its performance [49]. In 2009, Riihonen *et al.* investigated dual-hop full-duplex relay networks with AF/DF protocols and co-channel loop interference [50]. Ju *et al.* analyzed the bit error rate (BER) and achievable rate of dual-hop full-duplex relay networks where direct path is not available [51]. In 2012, Ng *et al.* investigated a joint optimization problem for resource allocation and scheduling in full-duplex multiple-input multiple-output orthogonal frequency division multiple access (MIMO-OFDMA) relay networks with AF and DF protocols [52]. Krikidis *et al.* investigated an optimal relay selection scheme that maximizes the instantaneous channel capacity and requires global channel state information (CSI). Also, they investigated several sub-optimal relay selection schemes that use partial CSI such as a) source-relay and relay-destination links, b) loop interference, c) source-relay links and loop interference [53]. Tabataba *et al.* derived achievable rates for AF-based full-duplex relay networks using analog network coding with channel estimation errors as well as information rate cut-set bounds in traditional routing with channel estimation errors [54]. In 2014, Choi *et al.* investigated two-way full-duplex relaying with a residual loop interference and imperfect channel state information (CSI), and derived the closed-form expressions for the outage probability [55].

1.2 Outline of Dissertation

In this dissertation, we consider the wireless relay networks.

In Chapter 2, we propose the proactive and the reactive relay selection schemes based on CDFs of SNRs for one-way relay networks over Nakagami-m fading channels. We analyze average relay fairness by deriving relay selection probability. For the proactive CDF-based relay selection scheme, we analyze diversity order by deriving the integral and asymptotic expressions for outage probability. For the reactive CDF-based relay selection scheme, we obtain diversity order by deriving the exact and asymptotic expressions for outage probability. Analytical results are verified by Monte Carlo simulations. It is shown that the analytical results are in complete agreement with simulation results. It is shown that the proposed schemes guarantee strict fairness among relays, and diversity order depends on the number of relays and fading severity parameters.

In Chapter 3, we propose the proactive and the reactive relay selection schemes based on CDFs of SNRs for two-way relay networks over Nakagami-m fading channels. For the proactive CDF-based relay selection scheme, we analyze average relay fairness by deriving relay selection probability. Also, we analyze diversity order by deriving the integral and asymptotic expressions for outage probability. For the reactive CDF-based relay selection scheme, we analyze average relay fairness by deriving the exact integral and asymptotic expressions for relay selection probability. Also, we obtain diversity order by deriving the exact and asymptotic expressions for outage probability. Analytical results are verified by Monte Carlo simulations. It is shown that the analytical results are in complete agreement with simulation results. It is shown that the proposed schemes guarantee strict fairness among relays, and diversity order depends on the number of relays and fading severity parameters.

Finally, in Chapter 4, the conclusion, possible applications, and future work are drawn.

1.3 Notations

We use the following notation: $\mathrm{Re}[x]$ denotes the real part of x. $\mathrm{Im}[x]$ denotes the imaginary part of x. Also, we explain mathematical functions and definitions used throughout the dissertation.

Definition 1 (Gamma Function [61]). *The gamma function is defined as*

$$\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt \tag{1.1}$$

for $\mathrm{Re}[x] > 0$.

Definition 2 (Incomplete Gamma Function [61]). *The lower incomplete gamma function is defined as*

$$\gamma(\alpha, x) = \int_0^x e^{-t} t^{\alpha-1} dt \tag{1.2}$$

for $\mathrm{Re}[\alpha] > 0$. *The upper incomplete gamma function is defined as*

$$\Gamma(\alpha, x) = \int_x^\infty e^{-t} t^{\alpha-1} dt. \tag{1.3}$$

Definition 3 (Hypergeometric Function [61]). *For nonnegative integers p and q, the hypergeometric function is defined as*

$${}_pF_q(\alpha_1, \alpha_2, \cdots, \alpha_p; \beta_1, \beta_2, \cdot, \beta_q; z) = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k (\alpha_2)_k \cdots (\alpha_p)_k}{(\beta_1)_k (\beta_2)_k \cdots (\beta_q)_k} \frac{z^k}{k!} \tag{1.4}$$

where $(a)_k$ is the Pochhammer symbol defined as

$$(a)_k = a(a+1)\cdots(a+k-1) = \frac{\Gamma(a+k)}{\Gamma(a)}. \tag{1.5}$$

As a special case of the hypergeometric function, the Gauss hypergeometric function is given by

$$\begin{aligned} {}_2F_1(\alpha_1, \alpha_2; \beta; z) &= \sum_{k=0}^{\infty} \frac{(\alpha_1)_k (\alpha_2)_k}{(\beta)_k} \frac{z^k}{k!} \\ &= \frac{\Gamma(\beta)}{\Gamma(\alpha_1)\Gamma(\alpha_2)} \sum_{k=0}^{\infty} \frac{\Gamma(\alpha_1 + k)\Gamma(\alpha_2 + k)}{\Gamma(beta + k)} \frac{z^k}{k!}. \end{aligned} \tag{1.6}$$

Definition 4 (Bessel Function [62]). *Bessel functions $Z_v(z)$ are solutions of the Bessel's differential equation*

$$\frac{d^2 Z_v}{dz^2} + \frac{1}{z}\frac{dZ_v}{dz} + \left(1 - \frac{v^2}{z^2}\right) Z_v = 0 \tag{1.7}$$

for an arbitrary complex number of v.

As a special case of Bessel functions, the Bessel functions of the first kind are solutions of Bessel's differential equation that are finite at the origin for integer or positive v, and diverge as z approaches zero for negative non-integer v. The Bessel functions of the first kind are defined as

$$J_v(z) = \frac{z^v}{2^v}\sum_{k=0}^{\infty}(-1)^k \frac{z^{2k}}{2^{2k}k!\Gamma(v+k+1)} \tag{1.8}$$

for $|\arg z| < \pi$. As a special case of Bessel functions, the Bessel functions of the second kind, which are called as Neumann functions, are solutions of the Bessel's differential equation that have a singularity at the origin and are multi-valued. The Bessel functions of the second kind are defined as

$$Y_v(z) = \frac{1}{\sin v\pi}\{\cos v\pi J_v(z) - J_{-v}(z)\} \tag{1.9}$$

for non-integer v and $|\arg z| < \pi$. As a special case of Bessel functions, the Bessel functions of the third kind, which is called as Hankel's functions, are defined as

$$H_v^{(1)}(z) = J_v(z) + iY_v(z) \tag{1.10}$$

and

$$H_v^{(2)}(z) = J_v(z) - iY_v(z) \tag{1.11}$$

where i is the imaginary unit. The Bessel functions are valid even for complex arguments z, and a special case is that of a purely imaginary argument. In this case, the solutions to the Bessel equation are called as the modified Bessel functions of the first and second kind, and are defined as

$$I_v(z) = \left(\frac{1}{2}z\right)^2 \sum_{k=0}^{\infty} \frac{(\frac{1}{4}z^2)^k}{k!\Gamma(v+k+1)} \tag{1.12}$$

and

$$K_v(z) = \frac{1}{2}\pi \frac{I_{-v}(z) - I_v(z)}{\sin(v\pi)}, \tag{1.13}$$

respectively, for non-integer v. If v is an integer or zero, the right-hand side of these equations are replaced by its limiting value.

Table 1.1 and Table 1.2 list the abbreviations and symbols used throughout the dissertation, respectively.

Table 1.1. List of abbreviations

3GPP	third generation partnership project
AF	amplify-and-forward
AWGN	additive white Gaussian noise
CDF	cumulative distribution function
CP	control parameter
CSI	channel state information
CTS	clear-to-send
dB	decibels, $10\log_{10}(\cdot)$
DF	decode-and-forward
DMT	diversity-multiplexing tradeoff
D2D	device-to-device
i.i.d.	independent and identically distributed
i.ni.d.	independent and not identically distributed
LTE	long term evolution
MGF	moment generating function
PDF	probability density function
RTS	ready-to-send
SNR	signal-to-noise ratio
TWRN	two-way relay network
QoS	quality of service

Table 1.2. List of symbols

$\in$	is an element of
$\notin$	is not an element of
$[\cdot]$	closed interval
$\{x_n\}_{n=1}^{N}$	set of elements $x_1, x_2, \cdots, x_N$
$\arg(\cdot)$	argument
$e^{(\cdot)}$	exponential function
$\exp(\cdot)$	exponential function
$\max\{x_1, x_2\}$	maximum of x_1 and x_2
$\min\{x_1, x_2\}$	minimum of x_1 and x_2
$\Pr\{\cdot\}$	probability
∞	infinity
$\int_a^b (\cdot) dx$	definite integral
$\prod_{n=1}^{N}$	multiple product
$\sum_{n=1}^{N}$	multiple sum
$n!$	factorial
$\lvert \cdot \rvert$	absolute value
$=$	equal
$\neq$	not equal
$\approx$	approximately equal
$\geq$	greater than or equal to
$\leq$	less than or equal to
$>$	strictly greater than
$<$	strictly less than
$\gg$	much greater than
$\ll$	much less than

Chapter 2

Relay Selection Based on CDFs of SNRs for One-Way Relay Networks

Cooperative diversity is an efficient way to avoid wireless channel impairments and obtain spatial diversity by using relays [63]-[72]. Relays share their resources and assist each other to obtain the benefits of multiple-input-multiple-output (MIMO) systems for cooperative diversity. In a cooperative wireless network with multiple relays, one of the most useful techniques for cooperative diversity is relay selection. Since relay selection schemes are simple to implement practically and do not need strict time synchronization among relays, various relay selection schemes are investigated and analyzed [63]-[66]. In [63] and [64], simple outage optimal relay selection scheme, namely, *opportunistic relaying* is introduced where a relay is selected based on maximum end-to-end signal-to-noise ratio (SNR). It guarantees full diversity with low overhead for relays. In [65], relay selection scheme with buffers, namely, *max-max relay selection* is introduced. This scheme selects relays for reception and transmission, respectively, and achieves full diversity with additional SNR gain. In [66], the effect of outdated channel estimate on amplify-and-forward (AF) opportunistic relaying is investigated. However, most of previous works on relay selection have only focused on reliability at destination.

In practice, statistics of the channels are not identical since statistics of the channel between two nodes vary depending on their locations. When the statistics of channels are not identical, channel gain-based relay selection schemes lead to fairness problem by selecting some particular relays more frequently than other relays. Since this problem causes reduction of lifetime in the network where mobiles are used as relays and have limited battery power [67]-[69], it is needed to consider fairness among relays as well as reliability at destination [70]-[72]. In [70], the concept of physical-layer fairness is dealt in AF cooperative diversity systems by attributing a weight coefficient to each relay depending on its average channel state. In [71], power reward is used to select a relay and improve fairness for energy-constrained ad-hoc networks. In [72], reactive proportional fair relay selection is proposed.

Cumulative distribution function (CDF)-based scheduling is useful to achieve fairness as compared with other fairness-aware scheduling when the statistics of channels are not identical. A user is selected whose rate is the least probable to become higher, that is, whose value of CDF of rate is the highest. CDF-based scheduling is introduced by many works [73]-[84] with comparison of other scheduling. In [77] and [79], it is applied to serve heterogeneous networks to control the traffic. In [80], it is employed for multi-cell OFDMA networks. To the best of our knowledge, the relay selection adopting CDF-based scheduling for one-way relay networks where channels experience different fading has not been studied yet, but it is worth to study it due to its advantages.

We propose the proactive and the reactive relay selection schemes based on CDFs of SNRs for one-way relay networks over Nakagami-m fading channels. For both the proactive and the reactive relay selection schemes, we analyze average relay fairness by deriving relay selection probability. For the proactive relay selection scheme, we derive the integral and asymptotic expressions for outage probability. Also, for the reactive relay selection scheme, we derive the exact closed-form and asymptotic expressions for outage probability. Analytical results are verified by Monte Carlo simulations. The average relay fairness and the outage probability are compared with various relay selection schemes.

2.1 System Model

Consider an one-way relay network with a single source S, a single destination D, and the set of K potential relays, $\mathcal{R} = \{r_1, r_2, \cdots, r_K\}$. Assume that each node has a single antenna and all nodes do not transmit and receive signals simultaneously. Also assume that there is no direct path between source S and destination D, and decode-and-forward (DF) relaying is adopted.

The received signal at node j from node i is given by

$$y_j = h_{i,j} x_i + n_j \tag{2.1}$$

where $h_{i,j}$ is the fading channel coefficient between node i and node j, x_i is a transmitted signal from node i with transmit power P_i, and $n_j \sim \mathcal{CN}(0, N_0)$ is the additive white Gaussian noise (AWGN) at node j. Assume that the channel coefficient from node i to node j, $h_{i,j}$, $i \in \{S, r_1, r_2, \cdots, r_K\}$, $j \in \{r_1, r_2, \cdots, r_K, D\}$, follows Nakagami distribution with the PDF

$$f_{|h_{i,j}|}(y) = \frac{2m_{i,j}^{m_{i,j}}}{\Gamma(m_{i,j})\Omega_{i,j}^{m_{i,j}}} y^{2m_{i,j}-1} e^{-\frac{m_{i,j}}{\Omega_{i,j}} y^2}, \quad y \geq 0, \tag{2.2}$$

where $m_{i,j}$ is the fading severity parameter and $\Omega_{i,j}$ is average fading power. The Nakagami distribution can be used for modelling different types of fading channels, which are characterized by the associated parameter $m_{i,j}$. Specifically, the Rayleigh fading channel can be modelled by the Nakagami distribution with value of $m_{i,j} = 1$. The Rician and lognormal fading channels can be modelled by the Nakagami distribution with values of $m_{i,j} > 1$. Also, Nakagami distribution for $m_{i,j} = 1/2$ can model the one-side Gaussian fading channel, i.e., the worst-case fading condition. We consider a block fading model where all channel gains remain constant during the whole data transmission and change independently across different ones [85], [86]. Assume that all channels are reciprocal and have an AWGN with zero mean and variance N_0. The received SNR at node j by transmission of node i is given by

$$Z_{i,j} = |h_{i,j}|^2 \frac{P_i}{N_0}. \tag{2.3}$$

Assume $P_S = P_{r_1} = \cdots = P_{r_K} = P$. The PDF and CDF of the received SNR $Z_{i,j}$ are given by

$$f_{Z_{i,j}}(z) = \frac{1}{\Gamma(m_{i,j})} \left(\frac{m_{i,j}}{\bar{z}_{i,j}} \right)^{m_{i,j}} z^{m_{i,j}-1} e^{\frac{m_{i,j} z}{\bar{z}_{i,j}}}, \quad z \geq 0, \tag{2.4}$$

and

$$F_{Z_{i,j}}(z) = \frac{\gamma\left(m_{i,j}, \frac{m_{i,j} z}{\bar{z}_{i,j}}\right)}{\Gamma(m_{i,j})}, \quad z \geq 0, \tag{2.5}$$

respectively, where $\bar{z}_{i,j} = \Omega_{i,j} P / N_0$.

We consider both proactive and reactive relay selection depending on whether a relay is selected before or after the source S broadcasts.

If the relay is chosen before the source transmission, it is called as *proactive relay selection*. In this way, only the selected relay spends energy for reception and all the other relays which are not selected can avoid receiving the signal from the source by remaining idle. Thus, the proactive relay selection has an advantage of saving reception energy in the relays which are not selected [63], [64]. However, relay selection complexity is increased by the number of potential relays since all relays participate for relay selection [64]. On the other hand, performing relay selection after the source transmission is called as *reactive relay selection*. The reactive relay selection excludes the relays that do not successfully decode the received signal from the source, thus, the relay selection complexity can be decreased [64]. However, the reactive relay selection requires all relays to receive the signal from the source during the first hop and cooperation overhead in reception energy scales proportionally with the number of potential relays [64].

2.1.1 Proactive CDF-Based Relay Selection

When the received SNRs at the relay r_k and at the destination D have the values of z_{S,r_k} and $z_{r_k,D}$, respectively, a relay is selected such that

$$r^* = \arg\max_{r_k \in \mathcal{R}} \min\{F_{Z_{S,r_k}}(z_{S,r_k}),\ F_{Z_{r_k,D}}(z_{r_k,D})\} \tag{2.6}$$

where $F_{Z_{S,r_k}}(\cdot)$ and $F_{Z_{r_k,D}}(\cdot)$ are CDFs of Z_{S,r_k} and $Z_{r_k,D}$, respectively.

The relays overhear a single transmission of a ready-to-send (RTS) packet from the source and a clear-to-send (CTS) packet from the destination. From these packets, the relays assess how appropriate each of them is for relaying. The transmission of RTS packet from the source allows for the estimation of the instantaneous wireless channel between the source and the relay, at each relay. Similarly, the transmission of CTS packet from the destination allows for the estimation of the instantaneous wireless channel between the relay and the destination at each relay according to the reciprocity theorem [3]. Note that the source does not need to listen to the CTS packet from the destination. Since communication among all relays should be minimized to reduce overall overhead, a timer-based method is selected [63]: As soon as each relay receives the RTS packet and the CTS packet, it starts a timer from a parameter based on value of CDF. The timer of the relay with the best end-to-end channel conditions will expire first. That relay transmits a short duration flag packet signaling its presence. All relays, while waiting for their timer to reduce to zero (i.e., to expire), are in listening mode. As soon as they hear another relay to flag its presence or forward information, they back off.

After a relay is selected, the source S transmits a signal to the destination D in two phases. In the first phase, the source S broadcasts a signal to all potential relays. The selected relay decodes the received signal and re-encodes it. In the second phase, the selected relay transmits the re-encoded signal to the destination D.

2.1.2 Reactive CDF-Based Relay Selection

Unlike the proactive relay selection, a relay is selected after the source S broadcasts. The source S transmits a signal to the destination D in two phases. In the first phase, the source S broadcasts a signal to all potential relays. All relays try to decode the received signal. Assume that if the received SNR at the relay is larger than SNR threshold z_{th}, the relay correctly decodes its received signal. The SNR threshold is given by

$$z_{th} = 2^{2R} - 1 \tag{2.7}$$

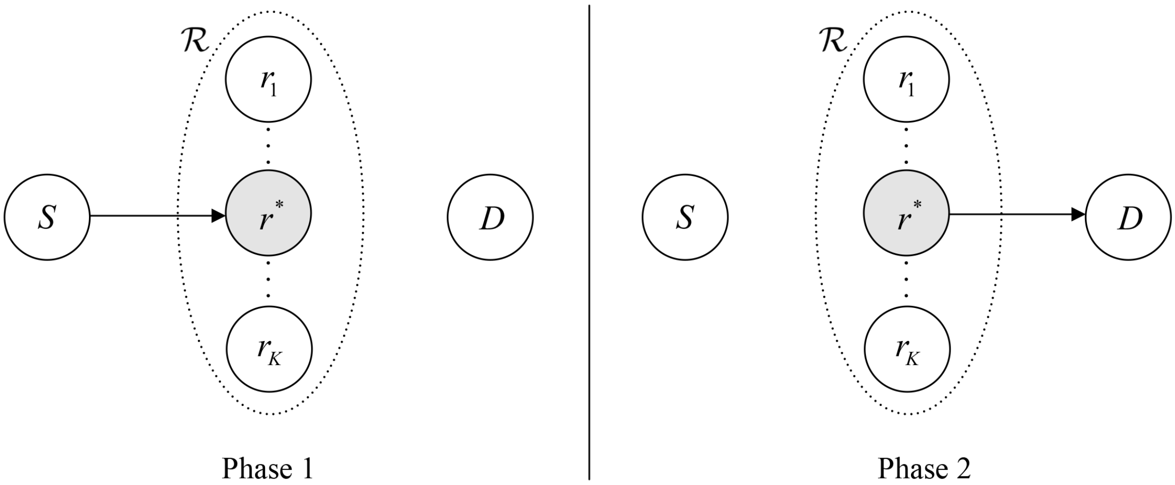
for the target rate R. This equation is derived from

$$R = \frac{1}{2}\log(1 + z_{th}) \tag{2.8}$$

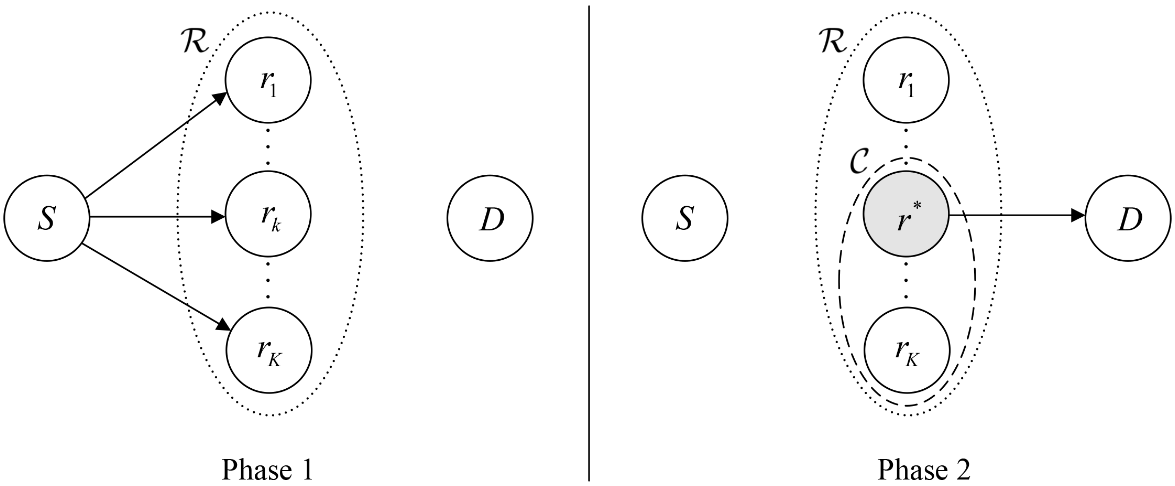
where the pre-log factor 1/2 comes from the fact that the source transmits its data to the destination over two phases. Let $\mathcal{C}$ denote the set of relays which decode the received signal successfully. When the received SNR at the destination D has the value of $z_{r_k,D}$, a relay is selected such that

$$r^* = \arg\max_{r_k \in \mathcal{C}} F_{Z_{r_k,D}}(z_{r_k,D}). \tag{2.9}$$

The selected relay re-encodes the received signal. In the second phase, the selected relay transmits the re-encoded signal to the destination D.



(a) Proactive relay selection



(b) Reactive relay selection

Figure 2.1. Proactive and reactive relay selection for one-way relay networks where the source S transmits information to the destination D with K relays. The shaded relay indicates the selected relay.

Relay selection can be done distributively by timer-based method [64] where the relays set timer whose duration is inversely proportional to a metric depending only on their CDFs. The relay with the shortest timer duration becomes the selected one and notifies others about its availability.

2.2 Performance Analysis of Proactive CDF-Based Relay Selection

2.2.1 Average Relay Fairness Analysis

We analyze the average relay fairness among all potential relays. Let ζ_{r_k} denote the ratio of the selection probability of r_k to the sum of selection probabilities of all potential relays, that is,

$$\zeta_{r_k} = \frac{\Pr(r^* = r_k)}{\sum_{l=1}^{K} \Pr(r^* = r_l)}. \tag{2.10}$$

Then, the average relay fairness of the network with K relays is given by [87]

$$\mathcal{F} = -\sum_{k=1}^{K} \zeta_{r_k} \frac{\log_2(\zeta_{r_k})}{\log_2(K)}. \tag{2.11}$$

Note that when strict fairness is achieved among relays, the average relay fairness $\mathcal{F}$ in (2.11) corresponds to 1. Since the power consumption is fixed for each relay when a relay is selected, the each selected number for transmission among relays is actually equal to the total power consumption of each relay. If strict fairness is not guaranteed, some relays run out of battery energy more rapidly than others, and the network becomes non-functional even when some relays have a large amount of battery-energy remaining. Therefore, average relay fairness could be important metric.

Since there is no case that a relay is not selected in the proactive relay selection, i.e., $\sum_{l=1}^{K} \Pr(r^* = r_l) = 1$, the ratio ζ_{r_k} is equal to the probability of selecting r_k. Let the random variables $U_{S,k}$ and $U_{k,D}$ be defined as $U_{S,k} \triangleq F_{Z_{S,r_k}}(Z_{S,r_k})$ and $U_{k,D} \triangleq F_{Z_{r_k,D}}(Z_{r_k,D})$, respectively. Note that although the channels are independent and non-identically distributed (i.ni.d.), $U_{S,k}$ and $U_{k,D}$ are independent and identically distributed (i.i.d.) uniform random variables ranging from 0 to 1 by *lemma 1* [73], [88].

Lemma 1. *Let X_i be a continuous random variable with CDF $F_{X_i}(x)$. Then, the transformed random variable $Y_i = F_{X_i}(X_i)$ is uniform random variable ranging from 0 to 1.*

Proof. The CDF of the random variable $F_{X_i}(X_i)$ is given by

$$\begin{aligned}\Pr(F_{X_i}(X_i) \leq y) &= \Pr\left(F_{X_i}^{-1}(F_{X_i}(X_i)) \leq F_{X_i}^{-1}(y)\right)\\ &= \Pr\left(X_i \leq F_{X_i}^{-1}(y)\right)\\ &= F_{X_i}\left(F_{X_i}^{-1}(y)\right)\\ &= y \end{aligned} \tag{2.12}$$

where $F_{X_i}^{-1}(\cdot)$ is the inverse function of CDF $F_{X_i}(\cdot)$. The first equality uses monotonicity of $F_{X_i}^{-1}(\cdot)$ and the third equality uses the definition of a CDF. It is proved that the random variable Y_i is uniformly distributed. ■

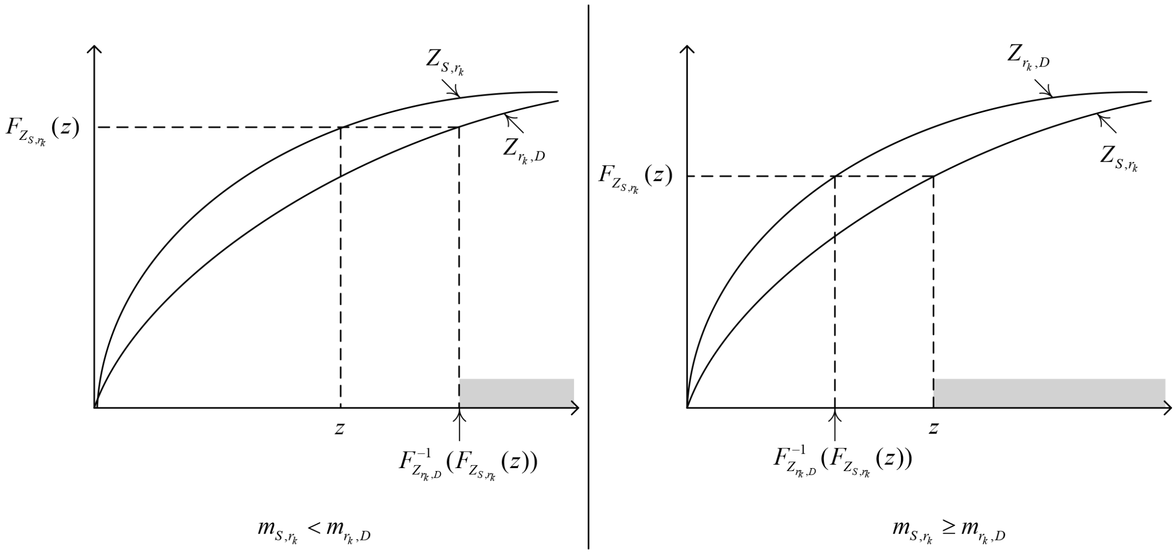
Define a random variable $U_k \triangleq \min\{U_{S,k}, U_{k,D}\}$. Note that U_k is the minimum of two i.i.d. uniform random variables and its CDF is given by [89]

$$\begin{aligned} F_{U_k}(u_k) &= 1 - (1 - F_{U_{S,k}}(u_k))(1 - F_{U_{k,D}}(u_k)) \\ &= 1 - (1 - u_k)^2. \end{aligned} \tag{2.13}$$

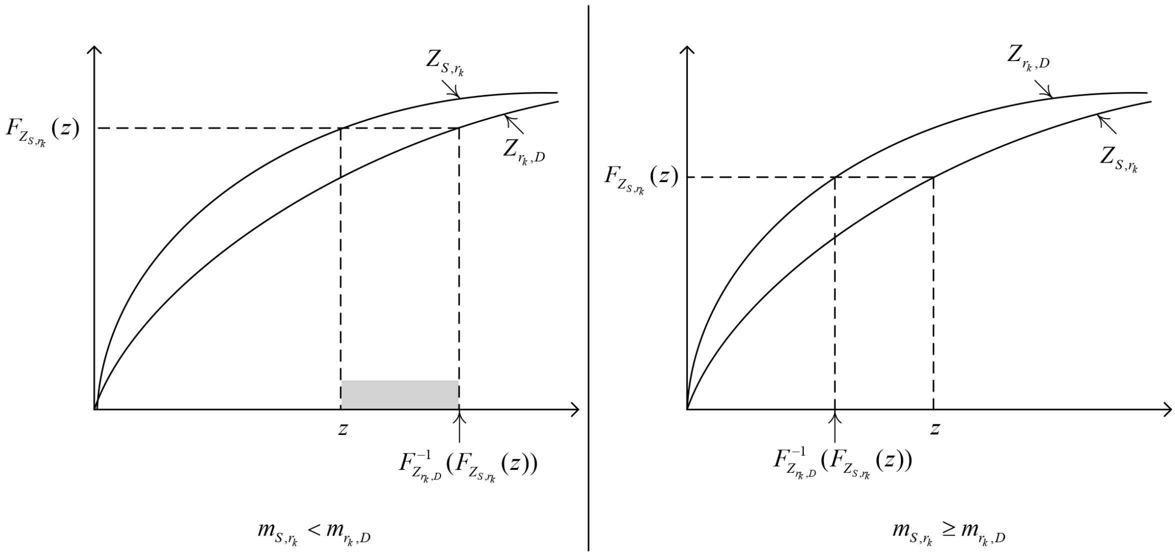
The probability of selecting r_k is given by

$$\begin{aligned} \Pr(r^* = r_k) &= \int_0^1 \Pr\left(U_j < u_k \text{ for all } j \neq k\right) f_{U_k}(u_k) du_k \\ &= \int_0^1 \left(\prod_{\substack{j=1 \\ j \neq k}}^{K} \Pr\left(U_j < u\right) \right) f_{U_k}(u_k) du_k \\ &= \int_0^1 \left(\prod_{\substack{j=1 \\ j \neq k}}^{K} F_{U_j}(u_k) \right) f_{U_k}(u_k) du_k \\ &= \int_0^1 (F_{U_k}(u_k))^{K-1} f_{U_k}(u_k) du_k \\ &= \frac{1}{K} \end{aligned} \tag{2.14}$$

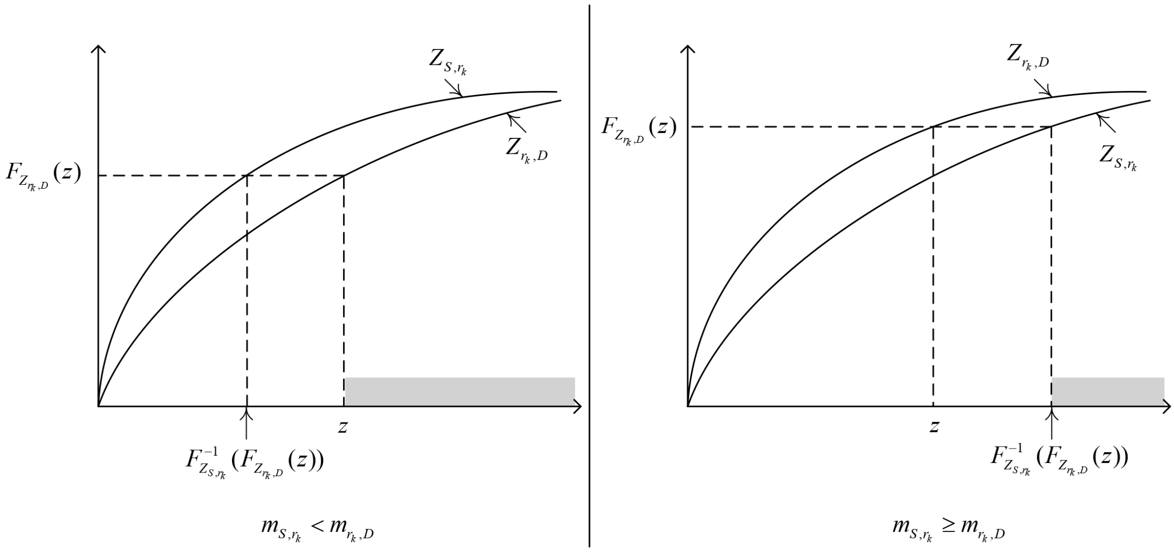
where $f_{U_k}(\cdot)$ is the PDF of U_k. The second and fourth equalities follow from the fact that U_k's are i.i.d. random variables and the third equality uses the definition of a CDF. Note that the probability of selecting the relay is related with the number of relays not the probability distribution of other relays. Since the probability of selecting the relay equals $1/K$, the ratio ζ_{r_k} in (2.10) becomes $1/K$ and the average relay fairness $\mathcal{F}$ in (2.11) becomes 1, which means that the proactive CDF-based relay selection scheme achieves strict fairness among relays.



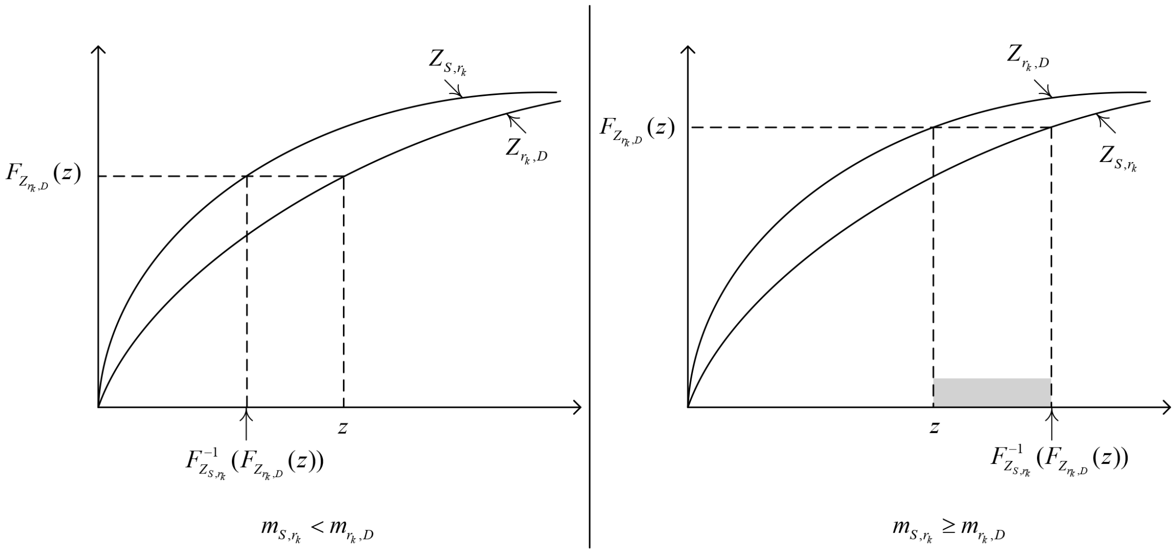
(a) The range of SNR for the case that $U_{S,k} < U_{k,D}$, $Z_{S,r_k} < Z_{r_k,D}$



(b) The range of SNR for the case that $U_{S,k} \geq U_{k,D}$, $Z_{S,r_k} < Z_{r_k,D}$



(c) The range of SNR for the case that $U_{S,k} \geq U_{k,D}$, $Z_{S,r_k} \geq Z_{r_k,D}$



(d) The range of SNR for the case that $U_{S,k} < U_{k,D}$, $Z_{S,r_k} \geq Z_{r_k,D}$

Figure 2.2. The range of SNR for the various cases in respect to the relations between $U_{S,k}$ and $U_{k,D}$ and between Z_{S,r_k} and $Z_{r_k,D}$. The shaded band indicates the range satisfying the conditions.

2.2.2 Outage Probability Analysis

An outage occurs when the end-to-end SNR at the destination D via the selected relay is smaller than the SNR threshold z_{th}. Define a random variable $Z_{r_k} \triangleq \min\{Z_{S,r_k}, Z_{r_k,D}\}$. Then, the outage probability is given by

$$
\begin{aligned}
P_{out} &= \sum_{k=1}^{K} \Pr(Z_{r_k} < z_{th},\, r^* = r_k) \\
&= \sum_{k=1}^{K} \int_0^{z_{th}} \Pr(r^* = r_k \,|\, Z_{r_k} = z) f_{Z_{r_k}}(z) dz \\
&= \sum_{k=1}^{K} I_k
\end{aligned}
\tag{2.15}
$$

where $f_{Z_{r_k}}(\cdot)$ is the PDF of Z_{r_k} and

$$
I_k = \int_0^{\gamma_{th}} \Pr(r^* = r_k \,|\, Z_{r_k} = z) f_{Z_{r_k}}(z) dz. \tag{2.16}
$$

By using the law of total probability, the probability of selecting r_k given that $Z_{r_k} = z$ is given by

$$
\begin{aligned}
\Pr(r^* = r_k \,|\, Z_{r_k} = z) \;=\; & \Pr(r^* = r_k,\, U_{S,k} < U_{k,D},\, Z_{S,r_k} < Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
& + \Pr(r^* = r_k,\, U_{S,k} \geq U_{k,D},\, Z_{S,r_k} < Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
& + \Pr(r^* = r_k,\, U_{S,k} \geq U_{k,D},\, Z_{S,r_k} \geq Z_{r_k,D} \,|\, Z_{r_k,D} = z) \\
& + \Pr(r^* = r_k,\, U_{S,k} < U_{k,D},\, Z_{S,r_k} \geq Z_{r_k,D} \,|\, Z_{r_k,D} = z).
\end{aligned}
\tag{2.17}
$$

The first term of the right-hand side of (2.17) can be written as

$$
\begin{aligned}
&\Pr(r^* = r_k,\, U_{S,k} < U_{k,D},\, Z_{S,r_k} < Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
&\quad = \prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(U_k \geq \min\{U_{S,i}, U_{i,D}\},\, U_{S,k} < U_{k,D},\, Z_{S,r_k} < Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
&\quad = \int_{\mathcal{A}_1} \prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(F_{Z_{S,r_k}}(z) \geq \min\{U_{S,i}, U_{i,D}\} \,|\, Z_{S,r_k} = z, Z_{r_k,D} = s) f_{Z_{r_k,D}}(s) ds
\end{aligned}
\tag{2.18}
$$

where $\mathcal{A}_1 = \{s \,|\, F_{Z_{S,r_k}}(z) < F_{Z_{r_k,D}}(s),\ z < s\}$. Since we can find the region $\mathcal{A}_1$ in Fig. 2.2(a), the first term of the right-hand side of (2.17) can be rewritten as

$$
\begin{aligned}
&\Pr(r^* = r_k,\, U_{S,k} < U_{k,D},\, Z_{S,r_k} < Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
&= \int_{\max\{z,\, a\}}^{\infty} \prod_{\substack{i=1 \\ i \neq k}}^{K} (1 - \Pr(F_{Z_{S,r_k}}(z) < \min\{U_{S,i}, U_{i,D}\} |\, Z_{S,r_k} = z, Z_{r_k,D} = s)) f_{Z_{r_k,D}}(s) ds \\
&= \int_{\max\{z,\, a\}}^{\infty} \prod_{\substack{i=1 \\ i \neq k}}^{K} (1 - \Pr(F_{Z_{S,r_k}}(z) < U_{S,i}, F_{Z_{S,r_k}}(z) < U_{i,D} \,|\, Z_{S,r_k} = z, Z_{r_k,D} = s)) \\
&\qquad \times f_{Z_{r_k,D}}(s) ds \\
&= \int_{\max\{z,\, a\}}^{\infty} \prod_{\substack{i=1 \\ i \neq k}}^{K} \{1 - (1 - F_{Z_{S,r_k}}(z))(1 - F_{Z_{S,r_k}}(z))\} f_{Z_{r_k,D}}(s) ds \\
&= \int_{\max\{z,\, a\}}^{\infty} \{1 - (1 - F_{Z_{S,r_k}}(z))^2\}^{K-1} f_{Z_{r_k,D}}(s) ds \\
&= \int_{\max\{z,\, a\}}^{\infty} \{2F_{Z_{S,r_k}}(z) - (F_{Z_{S,r_k}}(z))^2\}^{K-1} f_{Z_{r_k,D}}(s) ds
\end{aligned}
\tag{2.19}
$$

where $a = F^{-1}_{Z_{r_k,D}}(F_{Z_{S,r_k}}(z))$. The second term of the right-hand side of (2.17) can be written as

$$\begin{aligned}
&\Pr(r^* = r_k,\, U_{S,k} \geq U_{k,D},\, Z_{S,r_k} < Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
&\quad = \prod_{\substack{i=1 \\ i\neq k}}^{K} \Pr(U_k \geq \min\{U_{S,i}, U_{i,D}\},\, U_{S,k} \geq U_{k,D},\, Z_{S,r_k} < Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
&\quad = \int_{\mathcal{A}_2} \prod_{\substack{i=1 \\ i\neq k}}^{K} \Pr(F_{Z_{r_k,D}}(s) \geq \min\{U_{S,i}, U_{i,D}\} \,|\, Z_{S,r_k} = z, Z_{r_k,D} = s) f_{Z_{r_k,D}}(s) ds \qquad (2.20)
\end{aligned}$$

where $\mathcal{A}_2 = \{s \,|\, F_{Z_{S,r_k}}(z) \geq F_{Z_{r_k,D}}(s),\ z < s\}$. Since we can find the region $\mathcal{A}_2$ in Fig. 2.2(b), the second term of the right-hand side of (2.17) can be rewritten as

$$\begin{aligned}
&\Pr(r^* = r_k,\, U_{S,k} \geq U_{k,D},\, Z_{S,r_k} < Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
&= \int_z^{\max\{z,\, a\}} \prod_{\substack{i=1 \\ i\neq k}}^{K} (1 - \Pr(F_{Z_{r_k,D}}(s) < \min\{U_{S,i}, U_{i,D}\} |\, Z_{S,r_k} = z, Z_{r_k,D} = s)) f_{Z_{r_k,D}}(s) ds \\
&= \int_z^{\max\{z,\, a\}} \prod_{\substack{i=1 \\ i\neq k}}^{K} (1 - \Pr(F_{Z_{r_k,D}}(s) < U_{S,i}, F_{Z_{r_k,D}}(s) < U_{i,D} \,|\, Z_{S,r_k} = z, Z_{r_k,D} = s)) \\
&\qquad \times f_{Z_{r_k,D}}(s) ds \\
&= \int_z^{\max\{z,\, a\}} \prod_{\substack{i=1 \\ i\neq k}}^{K} \{1 - (1 - F_{Z_{r_k,D}}(s))(1 - F_{Z_{r_k,D}}(s))\} f_{Z_{r_k,D}}(s) ds \\
&= \int_z^{\max\{z,\, a\}} \{1 - (1 - F_{Z_{r_k,D}}(s))(1 - F_{Z_{r_k,D}}(s))\}^{K-1} f_{Z_{r_k,D}}(s) ds \\
&= \int_z^{\max\{z,\, a\}} \{2F_{Z_{r_k,D}}(s) - (F_{Z_{r_k,D}}(s))^2\}^{K-1} f_{Z_{r_k,D}}(s) ds. \qquad (2.21)
\end{aligned}$$

The third term of the right-hand side of (2.17) can be written as

$$\begin{aligned}
&\Pr(r^* = r_k,\, U_{S,k} \geq U_{k,D},\, Z_{S,r_k} \geq Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
&\quad = \int_{\mathcal{A}_3} \prod_{\substack{i=1 \\ i\neq k}}^{K} \Pr(F_{Z_{r_k,D}}(z) \geq \min\{U_{S,i}, U_{i,D}\} \,|\, Z_{S,r_k} = s, Z_{r_k,D} = z) f_{Z_{S,r_k}}(s) ds \qquad (2.22)
\end{aligned}$$

where $\mathcal{A}_3 = \{s \,|\, F_{Z_{S,r_k}}(z) \geq F_{Z_{r_k,D}}(s),\ z \geq s\}$. Similarly, since we can find the region $\mathcal{A}_3$ in Fig. 2.2(c), the third term of the right-hand side of (2.17) can be rewritten as

$$\begin{aligned}
&\Pr(r^* = r_k,\, U_{S,k} \geq U_{k,D},\, Z_{S,r_k} \geq Z_{r_k,D} \,|\, Z_{S,r_k} = z) \\
&\qquad = \int_{\max\{z,\, b\}}^{\infty} \{2F_{Z_{r_k,D}}(z) - (F_{Z_{r_k,D}}(z))^2\}^{K-1} f_{Z_{S,r_k}}(s) ds \qquad (2.23)
\end{aligned}$$

where $b = F^{-1}_{Z_{S,r_k}}(F_{Z_{r_k,D}}(z))$. The fourth term of the right-hand side of (2.17) can be written as

$$\begin{aligned}
&\Pr(r^* = r_k,\, U_{S,k} < U_{k,D},\, Z_{S,r_k} \geq Z_{r_k,D} \,|\, Z_{r_k,D} = z) \\
&\quad = \int_{\mathcal{A}_4} \prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(F_{Z_{r_k,D}}(s) \geq \min\{U_{S,i}, U_{i,D}\} \,|\, Z_{S,r_k} = s, Z_{r_k,D} = z) f_{Z_{S,r_k}}(s) ds \quad (2.24)
\end{aligned}$$

where $\mathcal{A}_4 = \{s \,|\, F_{Z_{S,r_k}}(z) < F_{Z_{r_k,D}}(s),\ z \geq s\}$. Similarly, since we can find the region $\mathcal{A}_4$ in Fig. 2.2(d), the fourth term of the right-hand side of (2.17) can be rewritten as

$$\begin{aligned}
&\Pr(r^* = r_k,\, U_{S,k} < U_{k,D},\, Z_{S,r_k} \geq Z_{r_k,D} \,|\, Z_{r_k,D} = z) \\
&\qquad = \int_{z}^{\max\{z,\, b\}} \{2F_{Z_{S,r_k}}(s) - (F_{Z_{S,r_k}}(s))^2\}^{K-1} f_{Z_{S,r_k}}(s) ds. \qquad (2.25)
\end{aligned}$$

From (2.16), (2.17), (2.19), (2.21), (2.23), and (2.25), I_k is given by

$$\begin{aligned}
I_k \;=\; &\int_0^{z_{th}} \int_{\max\{z,\, a\}}^{\infty} \{2F_{Z_{S,r_k}}(z) - (F_{Z_{S,r_k}}(z))^2\}^{K-1} f_{Z_{r_k,D}}(s) f_{Z_{S,r_k}}(z) ds dz \\
&+ \int_0^{z_{th}} \int_{z}^{\max\{z,\, a\}} \{2F_{Z_{r_k,D}}(s) - (F_{Z_{r_k,D}}(s))^2\}^{K-1} f_{Z_{r_k,D}}(s) f_{Z_{S,r_k}}(z) ds dz \\
&+ \int_0^{z_{th}} \int_{\max\{z,\, b\}}^{\infty} \{2F_{Z_{r_k,D}}(z) - (F_{Z_{r_k,D}}(z))^2\}^{K-1} f_{Z_{S,r_k}}(s) f_{Z_{r_k,D}}(z) ds dz \\
&+ \int_0^{z_{th}} \int_{z}^{\max\{z,\, b\}} \{2F_{Z_{S,r_k}}(s) - (F_{Z_{S,r_k}}(s))^2\}^{K-1} f_{Z_{S,r_k}}(s) f_{Z_{r_k,D}}(z) ds dz. \quad (2.26)
\end{aligned}$$

In the case that $m_{S,r_k} \geq m_{r_k,D}$, the first term on the right-hand side of (2.26) is given by

$$\begin{aligned}
\int_0^{z_{th}} \int_{\max\{z,\ a\}}^{\infty} & \{2F_{Z_{S,r_k}}(z) - (F_{Z_{S,r_k}}(z))^2\}^{K-1} f_{Z_{r_k,D}}(s) f_{Z_{S,r_k}}(z) ds dz \\
&= \int_0^{z_{th}} \{2F_{Z_{S,r_k}}(z) - (F_{Z_{S,r_k}}(z))^2\}^{K-1} f_{Z_{S,r_k}}(z)(1 - F_{Z_{r_k,D}}(z)) dz \\
&= \int_0^{z_{th}} \{2F_{Z_{S,r_k}}(z) - (F_{Z_{S,r_k}}(z))^2\}^{K-1} f_{Z_{S,r_k}}(z) dz \\
&\quad - \int_0^{z_{th}} \{2F_{Z_{S,r_k}}(z) - (F_{Z_{S,r_k}}(z))^2\}^{K-1} f_{Z_{S,r_k}}(z) F_{Z_{r_k,D}}(z) dz. \qquad (2.27)
\end{aligned}$$

The second term on the right-hand side of (2.26) is zero since $\max\{z,\ a\} = z$. The third term on the right-hand side of (2.26) is given by

$$\begin{aligned}
\int_0^{z_{th}} \int_{\max\{z,\ b\}}^{\infty} & \{2F_{Z_{r_k,D}}(z) - (F_{Z_{r_k,D}}(z))^2\}^{K-1} f_{Z_{S,r_k}}(s) f_{Z_{r_k,D}}(z) ds dz \\
&= \int_0^{z_{th}} \{2F_{Z_{r_k,D}}(z) - (F_{Z_{r_k,D}}(z))^2\}^{K-1} f_{Z_{r_k,D}}(z)(1 - F_{Z_{r_k,D}}(z)) dz \\
&= \int_0^{F_{Z_{r_k,D}}(z_{th})} (2t - t^2)^{K-1}(1-t) dt \\
&= \frac{1}{2K}\{2F_{Z_{r_k,D}}(z_{th}) - (F_{Z_{r_k,D}}(z_{th}))^2\}^K. \qquad (2.28)
\end{aligned}$$

The last term on the right-hand side of (2.26) is given by

$$\begin{aligned}
\int_0^{z_{th}} \int_z^{\max\{z,\ b\}} & \{2F_{Z_{S,r_k}}(s) - (F_{Z_{S,r_k}}(s))^2\}^{K-1} f_{Z_{S,r_k}}(s) f_{Z_{r_k,D}}(z) ds dz \\
&= \int_0^{z_{th}} \int_z^{b} \{2F_{Z_{S,r_k}}(s) - (F_{Z_{S,r_k}}(s))^2\}^{K-1} f_{Z_{S,r_k}}(s) f_{Z_{r_k,D}}(z) ds dz \\
&= \int_0^{z_{th}} \int_{F_{Z_{S,r_k}}(z)}^{F_{Z_{r_k,D}}(z)} (2t - t^2)^{K-1} dt f_{Z_{r_k,D}}(z) dz. \qquad (2.29)
\end{aligned}$$

From (2.26), (2.27), (2.28), and (2.29), we obtain the integral I_k as

$$I_k = \int_0^{F_{Z_{S,r_k}}(z_{th})} (2t - t^2)^{K-1} dt + \frac{1}{2K}\{2F_{Z_{r_k,D}}(z_{th}) - (F_{Z_{r_k,D}}(z_{th}))^2\}^K$$
$$- \int_0^{z_{th}} \{2F_{Z_{S,r_k}}(z) - (F_{Z_{S,r_k}}(z))^2\}^{K-1} f_{Z_{S,r_k}}(z) F_{Z_{r_k,D}}(z) dz$$
$$+ \int_0^{z_{th}} \int_{F_{Z_{S,r_k}}(z)}^{F_{Z_{r_k,D}}(z)} (2t - t^2)^{K-1} dt f_{Z_{r_k,D}}(z) dz. \quad (2.30)$$

When $\bar{z}_{i,j}$ is high, the incomplete gamma function can be approximated as [61, eq. (8.354.1)]

$$\gamma\left(m_{i,j}, \frac{m_{i,j}x}{\bar{z}_{i,j}}\right) \approx \frac{1}{m_{i,j}} \left(\frac{m_{i,j}x}{\bar{z}_{i,j}}\right)^{m_{i,j}}. \quad (2.31)$$

At high SNR region, the last term on the right-hand side of (2.30) goes to zero. Hence, I_k can be approximated as

$$I_k \approx \frac{1}{2K}\{2F_{Z_{S,r_k}}(z_{th}) - (F_{Z_{S,r_k}}(z_{th}))^2\}^K + \frac{1}{2K}\{2F_{Z_{r_k,D}}(z_{th}) - (F_{Z_{r_k,D}}(z_{th}))^2\}^K. \quad (2.32)$$

Similarly, in the case that $m_{S,r_k} < m_{r_k,D}$, I_k is approximated as

$$I_k \approx \frac{1}{2K}\{2F_{Z_{r_k,D}}(z_{th}) - (F_{Z_{r_k,D}}(z_{th}))^2\}^K + \frac{1}{2K}\{2F_{Z_{S,r_k}}(z_{th}) - (F_{Z_{S,r_k}}(z_{th}))^2\}^K. \quad (2.33)$$

Then, the outage probability is approximated as

$$P_{out} \approx \sum_{k=1}^{K} \left[\frac{1}{2K}\{2F_{Z_{S,r_k}}(z_{th}) - (F_{Z_{S,r_k}}(z_{th}))^2\}^K + \frac{1}{2K}\{2F_{Z_{r_k,D}}(z_{th}) - (F_{Z_{r_k,D}}(z_{th}))^2\}^K\right]. \quad (2.34)$$

From (2.31) and (2.34), the outage probability is approximated as

$$\begin{aligned}
P_{out} \approx & \frac{1}{2K} \sum_{k=1}^{K} \left[\left\{ \frac{1}{m_{S,r_k}\Gamma(m_{S,r_k})} \left(\frac{m_{S,r_k} z_{th}}{\bar{z}_{S,r_k}} \right)^{m_{S,r_k}} \left(2 - \frac{1}{m_{S,r_k}\Gamma(m_{S,r_k})} \left(\frac{m_{S,r_k} z_{th}}{\bar{z}_{S,r_k}} \right)^{m_{S,r_k}} \right) \right\}^{K} \right. \\
& \left. + \left\{ \frac{1}{m_{r_k,D}\Gamma(m_{r_k,D})} \left(\frac{m_{r_k,D} z_{th}}{\bar{z}_{r_k,D}} \right)^{m_{r_k,D}} \left(2 - \frac{1}{m_{r_k,D}\Gamma(m_{r_k,D})} \left(\frac{m_{r_k,D} z_{th}}{\bar{z}_{r_k,D}} \right)^{m_{r_k,D}} \right) \right\}^{K} \right] \\
\approx & \frac{1}{2K} \sum_{k=1}^{K} \left[\left\{ \frac{1}{m_{S,r_k}\Gamma(m_{S,r_k})} \left(\frac{m_{S,r_k} z_{th}}{\Omega_{S,r_k}\eta} \right)^{m_{S,r_k}} \left(\frac{2m_{S,r_k}\Gamma(m_{S,r_k})\Omega_{S,r_k}^{m_{S,r_k}} \eta^{m_{S,r_k}} - m_{S,r_k}^{m_{S,r_k}} z_{th}^{m_{S,r_k}}}{m_{S,r_k}\Gamma(m_{S,r_k})\Omega_{S,r_k}^{m_{S,r_k}} \eta^{m_{S,r_k}}} \right) \right\}^{K} \right. \\
& \left. + \left\{ \frac{1}{m_{r_k,D}\Gamma(m_{r_k,D})} \left(\frac{m_{r_k,D} z_{th}}{\Omega_{r_k,D}\eta} \right)^{m_{r_k,D}} \left(\frac{2m_{r_k,D}\Gamma(m_{r_k,D})\Omega_{r_k,D}^{m_{r_k,D}} \eta^{m_{r_k,D}} - m_{r_k,D}^{m_{r_k,D}} z_{th}^{m_{r_k,D}}}{m_{r_k,D}\Gamma(m_{r_k,D})\Omega_{r_k,D}^{m_{r_k,D}} \eta^{m_{r_k,D}}} \right) \right\}^{K} \right].
\end{aligned} \tag{2.35}$$

Since $2m_{x,y}\Omega_{x,y}^{m_{x,y}}\eta^{m_{x,y}} - m_{x,y}^{m_{x,y}} z_{th}^{m_{x,y}} \gg m_{x,y}\Omega_{x,y}^{m_{x,y}}\eta^{m_{x,y}}$, $x, y \in \{S, D, r_k\}$, at high SNR region, the outage probability can be rewritten as

$$\begin{aligned}
P_{out} \approx \frac{1}{2K} \sum_{k=1}^{K} & \left[\left\{ \frac{2}{m_{S,r_k}\Gamma(m_{S,r_k})} \left(\frac{m_{S,r_k} z_{th}}{\Omega_{S,r_k}\eta} \right)^{m_{S,r_k}} \right\}^{K} \right. \\
& \left. + \left\{ \frac{2}{m_{r_k,D}\Gamma(m_{r_k,D})} \left(\frac{m_{r_k,D} z_{th}}{\Omega_{r_k,D}\eta} \right)^{m_{r_k,D}} \right\}^{K} \right].
\end{aligned} \tag{2.36}$$

Let $m_{min} = \min\{m_{S,r_1}, \cdots, m_{S,r_K}, m_{r_1,D}, \cdots, m_{r_K,D}\}$. Then, the outage probability can be written as

$$\begin{aligned}
P_{out} \approx \frac{\eta^{-m_{min}K}}{2K} \sum_{k=1}^{K} & \left[\left\{ \frac{2}{m_{S,r_k}\Gamma(m_{S,r_k})\eta^{m_{S,r_k}-m_{min}}} \left(\frac{m_{S,r_k} z_{th}}{\Omega_{S,r_k}} \right)^{m_{S,r_k}} \right\}^{K} \right. \\
& \left. + \left\{ \frac{2}{m_{r_k,D}\Gamma(m_{r_k,D})\eta^{m_{r_k,D}-m_{min}}} \left(\frac{m_{r_k,D} z_{th}}{\Omega_{r_k,D}} \right)^{m_{r_k,D}} \right\}^{K} \right].
\end{aligned} \tag{2.37}$$

We can define the diversity order as [28]

$$d \triangleq \lim_{\eta \to \infty} -\frac{\log(P_{out})}{\log(\eta)}. \tag{2.38}$$

From (2.37) and (2.38), the diversity order d is obtained as

$$d = K \min\{m_{S,r_1}, \cdots, m_{S,r_K}, m_{r_1,D}, \cdots, m_{r_K,D}\}. \tag{2.39}$$

Note that the diversity order depends on the number of relays and the fading severity parameter $m_{i,j}$, $i \in \{S, r_1, \cdots, r_K\}$, $j \in \{r_1, \cdots, r_K, D\}$.

2.3 Performance Analysis of Reactive CDF-Based Relay Selection

2.3.1 Average Relay Fairness Analysis

There are $\binom{K-1}{l-1}$ subsets which have cardinality l and contain r_k among all subsets of $\mathcal{R}$. Let $\mathcal{R}_{l,n}^k$ denote the nth subset among them for $n = 1,\ 2,\ \cdots,\ \binom{K-1}{l-1}$. Then, the probability of selecting r_k is given by

$$\Pr(r^* = r_k) = \sum_{l=1}^{K} \sum_{n=1}^{\binom{K-1}{l-1}} \Pr(r^* = r_k \mid \mathcal{C} = \mathcal{R}_{l,n}^k)\Pr(\mathcal{C} = \mathcal{R}_{l,n}^k). \tag{2.40}$$

The probability that $\mathcal{C}$ becomes $\mathcal{R}_{l,n}^k$ is given by

$$\begin{aligned}\Pr(\mathcal{C} = \mathcal{R}_{l,n}^k) &= \Pr(r_k \in \mathcal{C}) \prod_{r_i \in \mathcal{R}_{l,n}^k \setminus \{r_k\}} \Pr(r_i \in \mathcal{C}) \prod_{r_j \notin \mathcal{R}_{l,n}^k} \Pr(r_j \notin \mathcal{C}) \\ &= \Pr(r_k \in \mathcal{C}) \prod_{r_i \in \mathcal{R}_{l,n}^k \setminus \{r_k\}} \Pr(r_i \in \mathcal{C}) \prod_{r_j \notin \mathcal{R}_{l,n}^k} \{1 - \Pr(r_j \in \mathcal{C})\}. \end{aligned} \tag{2.41}$$

The probability that r_i belongs to $\mathcal{C}$ is given by

$$\begin{aligned}\Pr(r_i \in \mathcal{C}) &= \Pr(Z_{S,r_i} \geq z_{th}) \\ &= 1 - \frac{\gamma\left(m_{S,r_i}, \frac{m_{S,r_i} z_{th}}{\bar{z}_{S,r_i}}\right)}{\Gamma(m_{S,r_i})}. \end{aligned} \tag{2.42}$$

The probability of selecting r_k given that $\mathcal{C}$ becomes $\mathcal{R}_{l,n}^k$ is given by

$$\begin{aligned}
\Pr(r^* = r_k \mid \mathcal{C} = \mathcal{R}_{l,n}^k) &= \int_0^1 \prod_{r_i \in \mathcal{R}_{l,n}^k \setminus \{r_k\}} \Pr\left(U_{i,D} < u\right) f_{U_{k,D}}(u) du \\
&= \int_0^1 \left(\prod_{\substack{i=1 \\ i \neq k}}^{l} F_{U_{i,D}}(u) \right) f_{U_{k,D}}(u) du \\
&= \frac{1}{l}. \qquad (2.43)
\end{aligned}$$

Plugging (2.41), (2.42), and (2.43) into (2.40), the probability of selecting r_k is given by

$$\begin{aligned}
\Pr(r^* = r_k) = \sum_{l=1}^{K} \sum_{n=1}^{\binom{K-1}{l-1}} &\left\{ \frac{1}{l} \frac{\Gamma\left(m_{S,r_k}, \frac{m_{S,r_k} z_{th}}{\bar{z}_{S,r_k}}\right)}{\Gamma(m_{S,r_k})} \prod_{r_i \in \mathcal{R}_{l,n}^k \setminus \{r_k\}} \frac{\Gamma\left(m_{S,r_i}, \frac{m_{S,r_i} z_{th}}{\bar{z}_{S,r_i}}\right)}{\Gamma(m_{S,r_i})} \right. \\
&\left. \times \prod_{r_j \notin \mathcal{R}_{l,n}^k} \frac{\gamma\left(m_{S,r_j}, \frac{m_{S,r_j} z_{th}}{\bar{z}_{S,r_j}}\right)}{\Gamma(m_{S,r_j})} \right\}. \qquad (2.44)
\end{aligned}$$

By using (2.44), the ratio ζ_{r_k} becomes

$$\zeta_{r_k} = \frac{\sum_{l=1}^{K} \sum_{n=1}^{\binom{K-1}{l-1}} \frac{1}{l} \frac{\Gamma\left(m_{S,r_k}, \frac{m_{S,r_k} z_{th}}{\bar{z}_{S,r_k}}\right)}{\Gamma(m_{S,r_k})} \prod_{r_i \in \mathcal{R}_{l,n}^k \setminus \{r_k\}} \frac{\Gamma\left(m_{S,r_i}, \frac{m_{S,r_i} z_{th}}{\bar{z}_{S,r_i}}\right)}{\Gamma(m_{S,r_i})} \prod_{r_j \notin \mathcal{R}_{l,n}^k} \frac{\gamma\left(m_{S,r_j}, \frac{m_{S,r_j} z_{th}}{\bar{z}_{S,r_j}}\right)}{\Gamma(m_{S,r_j})}}{\sum_{p=1}^{K} \sum_{l=1}^{K} \sum_{n=1}^{\binom{K-1}{l-1}} \frac{1}{l} \frac{\Gamma\left(m_{S,r_p}, \frac{m_{S,r_p} z_{th}}{\bar{z}_{S,r_p}}\right)}{\Gamma(m_{S,r_p})} \prod_{r_p \in \mathcal{R}_{l,n}^p \setminus \{r_p\}} \frac{\Gamma\left(m_{S,r_i}, \frac{m_{S,r_i} z_{th}}{\bar{z}_{S,r_i}}\right)}{\Gamma(m_{S,r_i})} \prod_{r_i \notin \mathcal{R}_{l,n}^p} \frac{\gamma\left(m_{S,r_j}, \frac{m_{S,r_j} z_{th}}{\bar{z}_{S,r_j}}\right)}{\Gamma(m_{S,r_j})}}. \qquad (2.45)$$

When $\bar{z}_{i,j}$ is high, the incomplete gamma function can be approximated as [61, eq. (8.354.1)]

$$\gamma\left(m_{i,j}, \frac{m_{i,j} x}{\bar{z}_{i,j}}\right) \approx \frac{1}{m_{i,j}} \left(\frac{m_{i,j} x}{\bar{z}_{i,j}}\right)^{m_{i,j}}. \qquad (2.46)$$

From (2.5) and (2.46), it is clear that as $\bar{z}_{i,j}$ increases, $\Pr(r_i \in \mathcal{C})$ goes to 1 and the decoding set becomes potential relay set $\mathcal{R}$. Then, the ratio ζ_{r_k} goes to $1/K$ and the average relay fairness $\mathcal{F}$ becomes 1, which means that the reactive CDF-based relay selection scheme achieves strict fairness among relays.

2.3.2 Outage Probability Analysis

An outage occurs when the received SNR at the destination D from the selected relay is smaller than the SNR threshold z_{th} or the decoding set is empty. The outage probability can be written as

$$\begin{aligned} P_{out} &= \sum_{k=1}^{K} \int_0^{z_{th}} \Pr(r^* = r_k \mid Z_{r_k,D} = z) f_{Z_{r_k,D}}(z)dz + \prod_{i=1}^{K} \Pr(Z_{S,r_i} < z_{th}) \\ &= \sum_{k=1}^{K} \int_0^{z_{th}} \sum_{m=1}^{K} \sum_{n=1}^{\binom{K-1}{m-1}} \left\{ \Pr(r^* = r_k | Z_{r_k,D} = z, \mathcal{C} = R_{m,n}^k) \Pr(\mathcal{C} = \mathcal{R}_{m,n}^k) \right\} f_{Z_{r_k,D}}(z)dz \\ &\quad + \prod_{i=1}^{K} \frac{\gamma\left(m_{S,r_i}, \frac{m_{S,r_i} z_{th}}{\bar{z}_{S,r_i}}\right)}{\Gamma(m_{S,r_i})} \\ &= \sum_{k=1}^{K} \sum_{l=1}^{K} \sum_{n=1}^{\binom{K-1}{l-1}} \Pr(\mathcal{C} = \mathcal{R}_{l,n}^k) \int_0^{z_{th}} \Pr(r^* = r_k \mid Z_{r_k,D} = z, \mathcal{C} = R_{l,n}^k) f_{Z_{r_k,D}}(z)dz + P_{nc} \end{aligned} \tag{2.47}$$

where P_{nc} is the probability that all potential relays do not decode the received signal successfully, which is given by

$$P_{nc} = \prod_{i=1}^{K} \frac{\gamma\left(m_{S,r_i}, \frac{m_{S,r_i} z_{th}}{\bar{z}_{S,r_i}}\right)}{\Gamma(m_{S,r_i})}. \tag{2.48}$$

The probability of selecting r_k given that the received SNR of the relay r_k is z and the decoding set $\mathcal{C}$ becomes the subset $R^k_{l,n}$ is given by

$$\begin{aligned}
\Pr(r^* = r_k \mid Z_{r_k,D} = z, \mathcal{C} = R^k_{l,n}) &= \prod_{\substack{j=1 \\ j\neq k, r_j\in\mathcal{C}, r_k\in\mathcal{C}}}^{l} \Pr(U_{j,D} < F_{Z_{r_k,D}}(z)) \\
&= \prod_{\substack{j=1 \\ j\neq k, r_j\in\mathcal{C}, r_k\in\mathcal{C}}}^{l} F_{U_{j,D}}(F_{Z_{r_k,D}}(z)) \\
&= (F_{U_{k,D}}(F_{Z_{r_k,D}}(z)))^{l-1} \\
&= (F_{Z_{r_k,D}}(z))^{l-1}. \qquad (2.49)
\end{aligned}$$

Plugging (2.49) into (2.47), the outage probability is rewritten as

$$\begin{aligned}
P_{out} &= \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{n=1}^{\binom{K-1}{l-1}} \Pr(\mathcal{C} = \mathcal{R}^k_{l,n}) \int_0^{z_{th}} (F_{Z_{r_k,D}}(z))^{l-1} f_{Z_{r_k,D}}(z)dz + P_{nc} \\
&= \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{n=1}^{\binom{K-1}{l-1}} \frac{1}{l}\Pr(\mathcal{C} = \mathcal{R}^k_{l,n})(F_{Z_{r_k,D}}(z_{th}))^{l} + P_{nc} \qquad (2.50)
\end{aligned}$$

where $\Pr(\mathcal{C} = \mathcal{R}^k_{l,n})$ is given in (2.41).

At high SNR region, by using the approximation of the incomplete gamma function in (2.46), the probability that the decoding set is empty is approximated as

$$P_{nc} \approx \prod_{i=1}^{K} \frac{1}{m_{S,r_i}\Gamma(m_{S,r_i})}\left(\frac{m_{S,r_i} z_{th}}{\Omega_{S,r_i}\eta}\right)^{m_{S,r_i}}. \qquad (2.51)$$

Since $\Pr(r_i \in \mathcal{C})$ goes to 1, the probability that $\mathcal{C}$ becomes $\mathcal{R}^k_{l,n}$ is given by

$$\begin{aligned}
\Pr(\mathcal{C} = \mathcal{R}^k_{l,n}) &\approx \prod_{r_i\in\mathcal{R}^k_{l,n}} \left\{1 - \frac{1}{m_{S,r_i}\Gamma(m_{S,r_i})}\left(\frac{m_{S,r_i} z_{th}}{\Omega_{S,r_i}\eta}\right)^{m_{S,r_i}}\right\} \\
&\quad \times \prod_{r_j\notin\mathcal{R}^k_{l,n}} \frac{1}{m_{S,r_j}\Gamma(m_{S,r_j})}\left(\frac{m_{S,r_j} z_{th}}{\Omega_{S,r_j}\eta}\right)^{m_{S,r_j}} \\
&\approx \prod_{r_j\notin\mathcal{R}^k_{l,n}} \frac{1}{m_{S,r_j}\Gamma(m_{S,r_j})}\left(\frac{m_{S,r_j} z_{th}}{\Omega_{S,r_j}\eta}\right)^{m_{S,r_j}}. \qquad (2.52)
\end{aligned}$$

Plugging (2.51) and (2.52) into (2.50), the outage probability can be written as

$$P_{out} \approx \sum_{k=1}^{K}\sum_{l=1}^{K}\sum_{n=1}^{\binom{K-1}{l-1}} \frac{1}{l}\left\{\frac{1}{m_{r_k,D}\Gamma(m_{r_k,D})}\left(\frac{m_{r_k,D}z_{th}}{\Omega_{r_k,D}\eta}\right)^{m_{r_k,D}}\right\}^{l}$$
$$\times \prod_{r_j\notin\mathcal{R}_{l,n}^{k}} \frac{1}{m_{S,r_j}\Gamma(m_{S,r_j})}\left(\frac{m_{S,r_j}z_{th}}{\Omega_{S,r_j}\eta}\right)^{m_{S,r_j}} + \prod_{i=1}^{K}\frac{1}{m_{S,r_i}\Gamma(m_{S,r_i})}\left(\frac{m_{S,r_i}z_{th}}{\Omega_{S,r_i}\eta}\right)^{m_{S,r_i}}. \tag{2.53}$$

At high SNR region, the decoding set becomes potential relay set $\mathcal{R}$, we can simplify the outage probability as

$$\begin{aligned} P_{out} &\approx \frac{1}{K}\sum_{k=1}^{K}\left[\left\{\frac{1}{m_{r_k,D}\Gamma(m_{r_k,D})}\left(\frac{m_{r_k,D}z_{th}}{\Omega_{r_k,D}\eta}\right)^{m_{r_k,D}}\right\}^{K}\right] \\ &\quad + \prod_{i=1}^{K}\frac{1}{m_{S,r_i}\Gamma(m_{S,r_i})}\left(\frac{m_{S,r_i}z_{th}}{\Omega_{S,r_i}\eta}\right)^{m_{S,r_i}} \\ &\approx \frac{1}{K}\sum_{k=1}^{K}\left[\eta^{-m_{r_k,D}K}\left\{\frac{1}{m_{r_k,D}\Gamma(m_{r_k,D})}\left(\frac{m_{r_k,D}z_{th}}{\Omega_{r_k,D}}\right)^{m_{r_k,D}}\right\}^{K}\right] \\ &\quad + \eta^{-\sum_{i=1}^{K}m_{S,r_i}}\prod_{i=1}^{K}\frac{1}{m_{S,r_i}\Gamma(m_{S,r_i})}\left(\frac{m_{S,r_i}z_{th}}{\Omega_{S,r_i}}\right)^{m_{S,r_i}}. \end{aligned} \tag{2.54}$$

Let $m_{min} = \min\{K\min\{m_{r_1,D}, m_{r_2,D}, \cdots, m_{r_K,D}\}, \sum_{i=1}^{K} m_{S,r_i}\}$. Then, the outage probability can be rewritten as

$$\begin{aligned} P_{out} \approx \frac{\eta^{-m_{min}}}{K} &\left[\sum_{k=1}^{K}\eta^{m_{min}-m_{r_k,D}K}\left\{\frac{1}{m_{r_k,D}\Gamma(m_{r_k,D})}\left(\frac{m_{r_k,D}z_{th}}{\Omega_{r_k,D}}\right)^{m_{r_k,D}}\right\}^{K}\right. \\ &\left. + \eta^{m_{min}-\sum_{i=1}^{K}m_{S,r_i}}\prod_{i=1}^{K}\frac{1}{m_{S,r_i}\Gamma(m_{S,r_i})}\left(\frac{m_{S,r_i}z_{th}}{\Omega_{S,r_i}}\right)^{m_{S,r_i}}\right]. \end{aligned} \tag{2.55}$$

From (2.38) and (2.55), the diversity order d is obtained as

$$d = \min\left\{K\min\{m_{r_1,D}, m_{r_2,D}, \cdots, m_{r_K,D}\}, \sum_{i=1}^{K} m_{S,r_i}\right\}. \tag{2.56}$$

Note that the diversity order depends on the number of relays and the fading severity parameter m_{S,r_i}, $i = 1, 2, \cdots, K$.

2.4 Numerical Results

Consider an one-way relay network consisting of a single source, a single destination, and various number of relays. we will use the notations $\mathbf{m}_{S,r_k} = \{m_{S,r_1}, m_{S,r_2}, \cdots, m_{S,r_K}\}$ and $\mathbf{m}_{r_k,D} = \{m_{r_1,D}, m_{r_2,D}, \cdots, m_{r_K,D}\}$. Assume that the noise variance N_0 is 1. Also, we assume that $\Omega_{i,j} = \mathrm{d}_{i,j}^{-3}$ where $\mathrm{d}_{i,j}$ is the distance between node i and node j. To compare the performance of the proposed schemes, channel gain based relay selection and proportional fair relay selection (which is relay selection based on the relative instantaneous-to-average value of SNR) are presented.

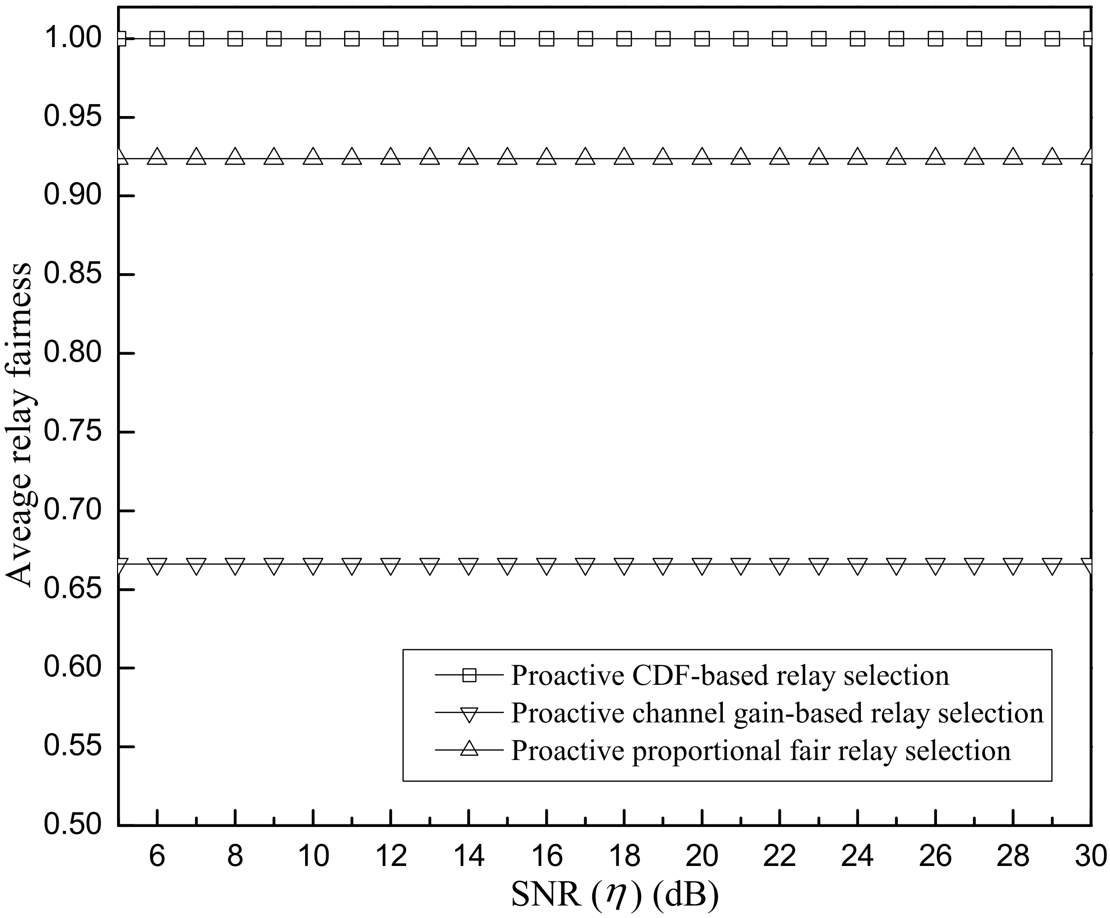
2.4.1 Average Relay Fairness

Fig. 2.3 shows the average relay fairness of various proactive relay selection schemes with various fading severity parameters for $K = 3$. It is shown that the proactive CDF-based relay selection scheme achieves 1 regardless of the SNR. It is shown that the proactive CDF-based relay selection scheme achieves higher average fairness than any other relay selection schemes.

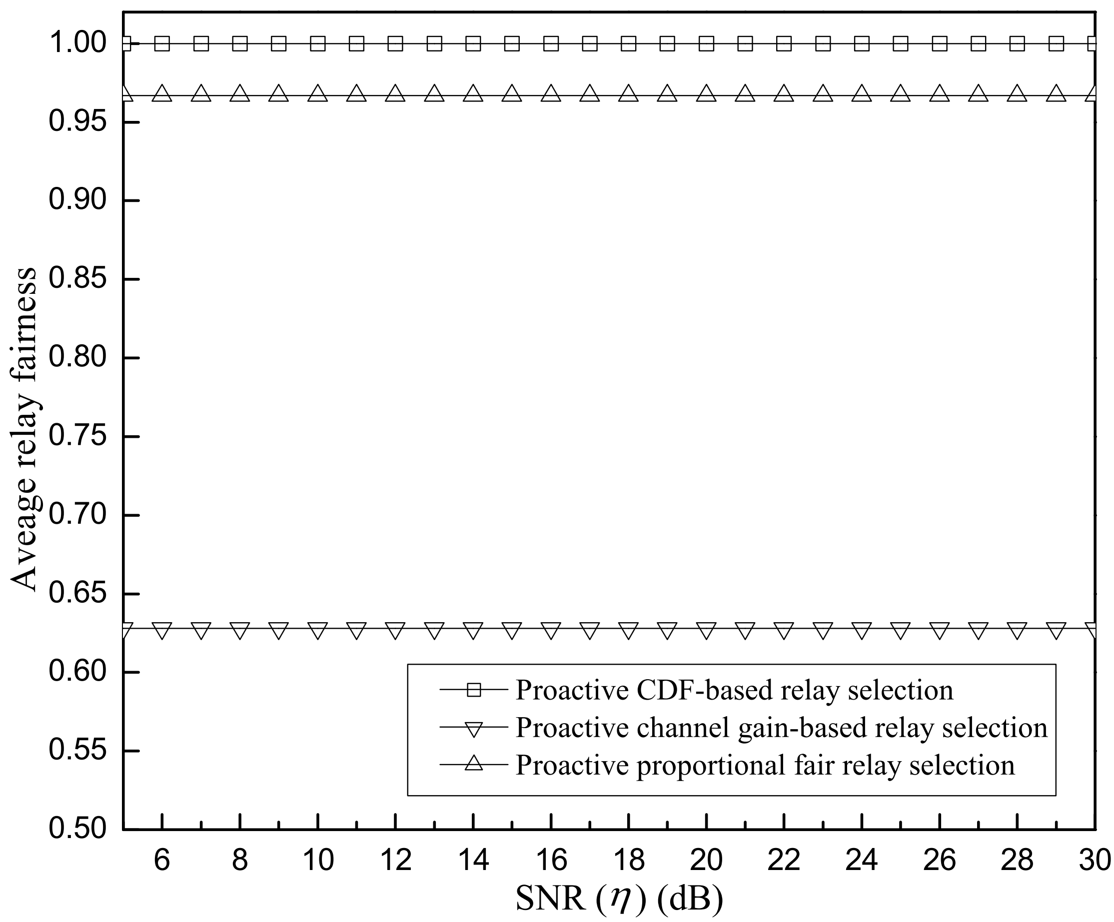
Fig. 2.4 show the average relay fairness of reactive CDF-based relay selection scheme with various fading severity parameters for $K = 3$. For simplicity, we will use the notation $m_{S,r_k} = m_{r_k,D} = m$ in Fig. 2.4(a). It is shown that the analytical results of the reactive CDF-based relay selection scheme perfectly match the simulation results. Fig. 2.4(a) shows that as the value of parameter decreases, the average relay fairness of reactive CDF-based relay selection scheme goes to 1 quickly. Fig. 2.4(b) shows that for $\eta \geq 10$ dB, the higher average relay fairness is achieved when $\mathbf{m}_{S,r_k} = \{1, 2, 3\}$

and $\mathbf{m}_{r_k,D} = \{1, 2, 3\}$.

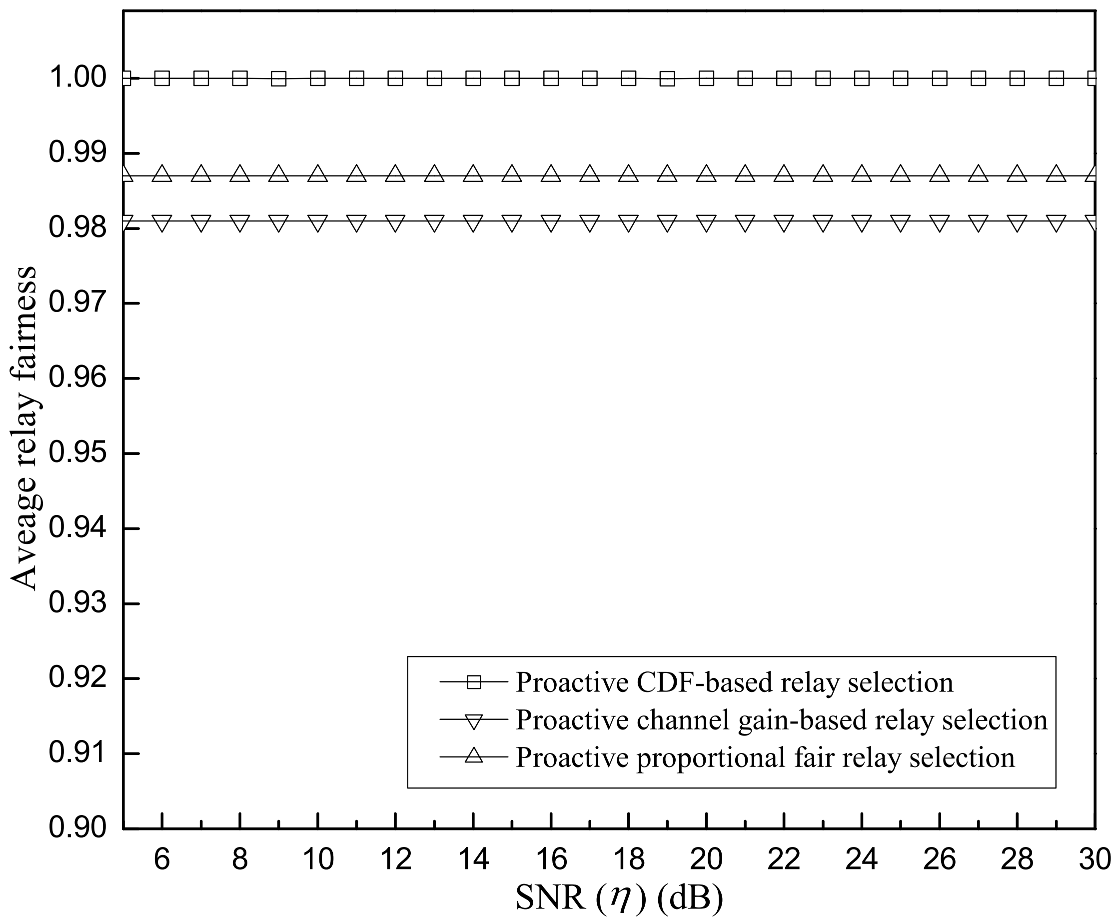
Fig. 2.5 shows the average relay fairness of various reactive relay selection schemes with various fading severity parameters for $K = 3$. It is shown that the reactive CDF-based relay selection scheme achieves higher average relay fairness than the reactive channel gain-based relay selection scheme and the reactive proportional fair relay selection scheme. It is shown that as the SNR increases, the average relay fairness of reactive CDF-based relay selection scheme goes to 1.



(a) $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{0.5, 1.0, 2.0\}$

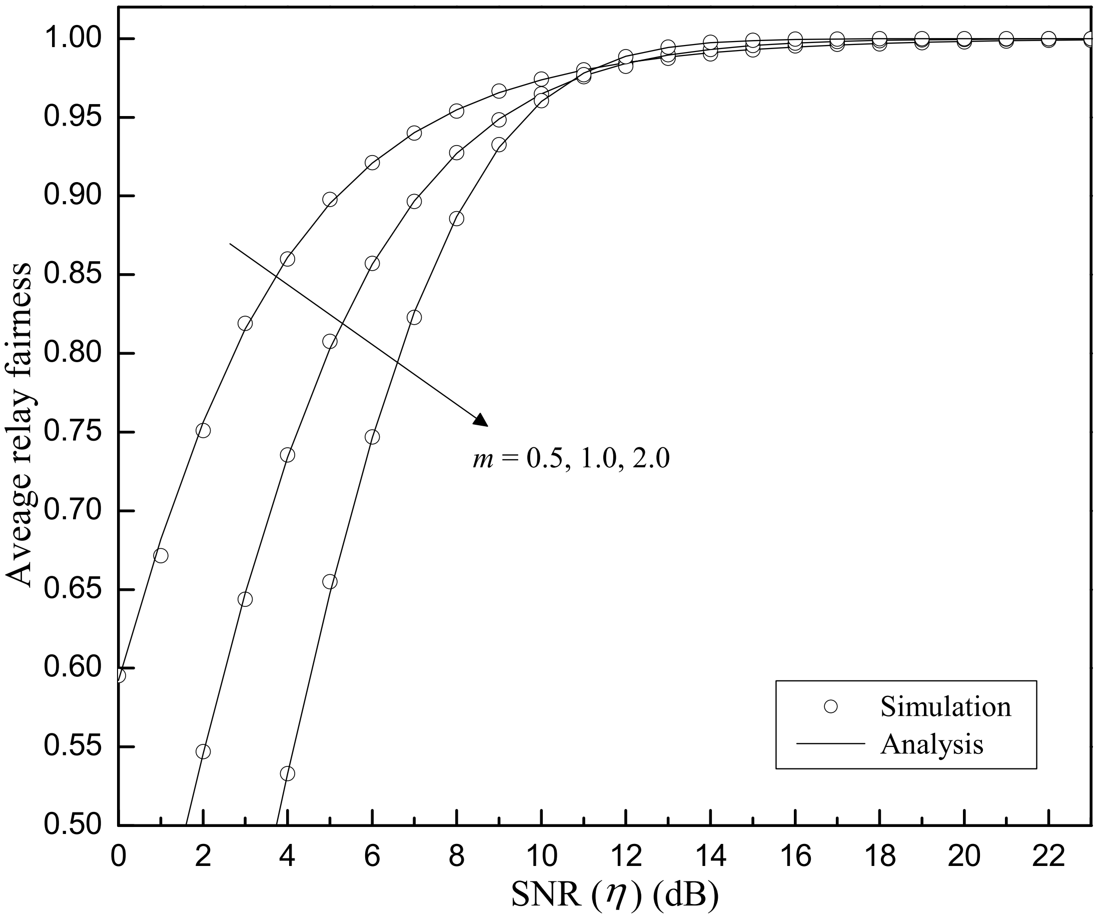


(b) $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{1.0, 2.0, 3.0\}$



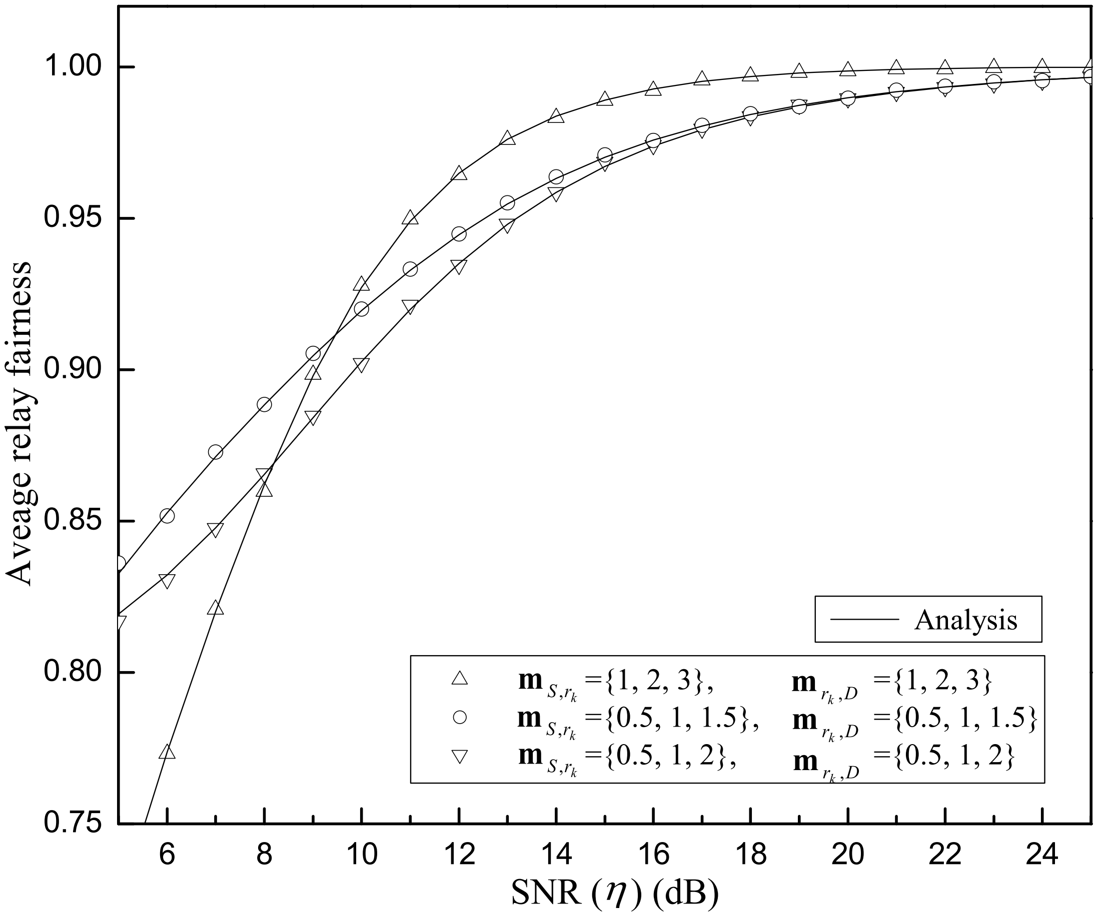
(c) $\mathbf{m}_{S,r_k} = \{0.5, 1.0, 1.5\}$ and $\mathbf{m}_{r_k,D} = \{3.0, 2.0, 0.5\}$

Figure 2.3. Average relay fairness of various proactive relay selection schemes.



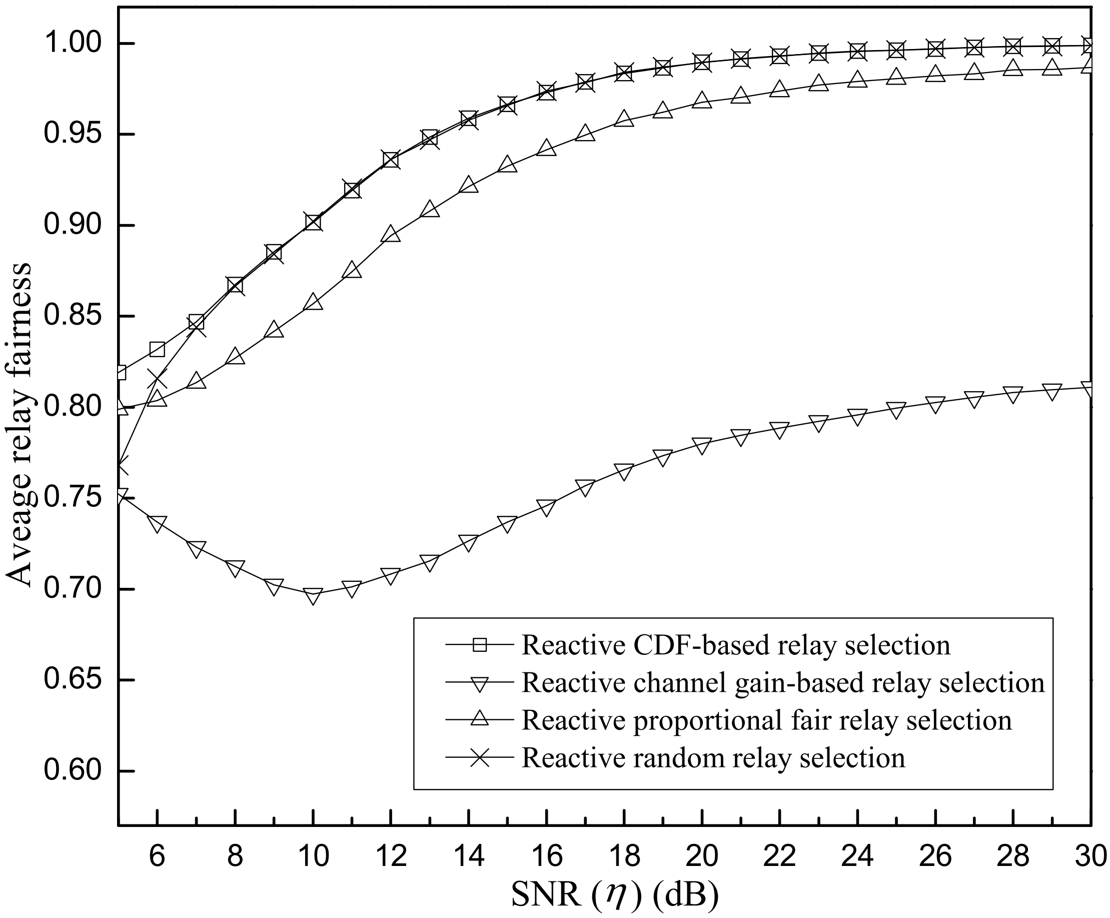
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
0.55
0.50
Aveage relay fairness
0
2
4
6
8
10
12
14
16
18
20
22
SNR (η) (dB)
m = 0.5, 1.0, 2.0
Simulation
Analysis

(a) $m_{S,r_k} = m_{r_k,D} = m$

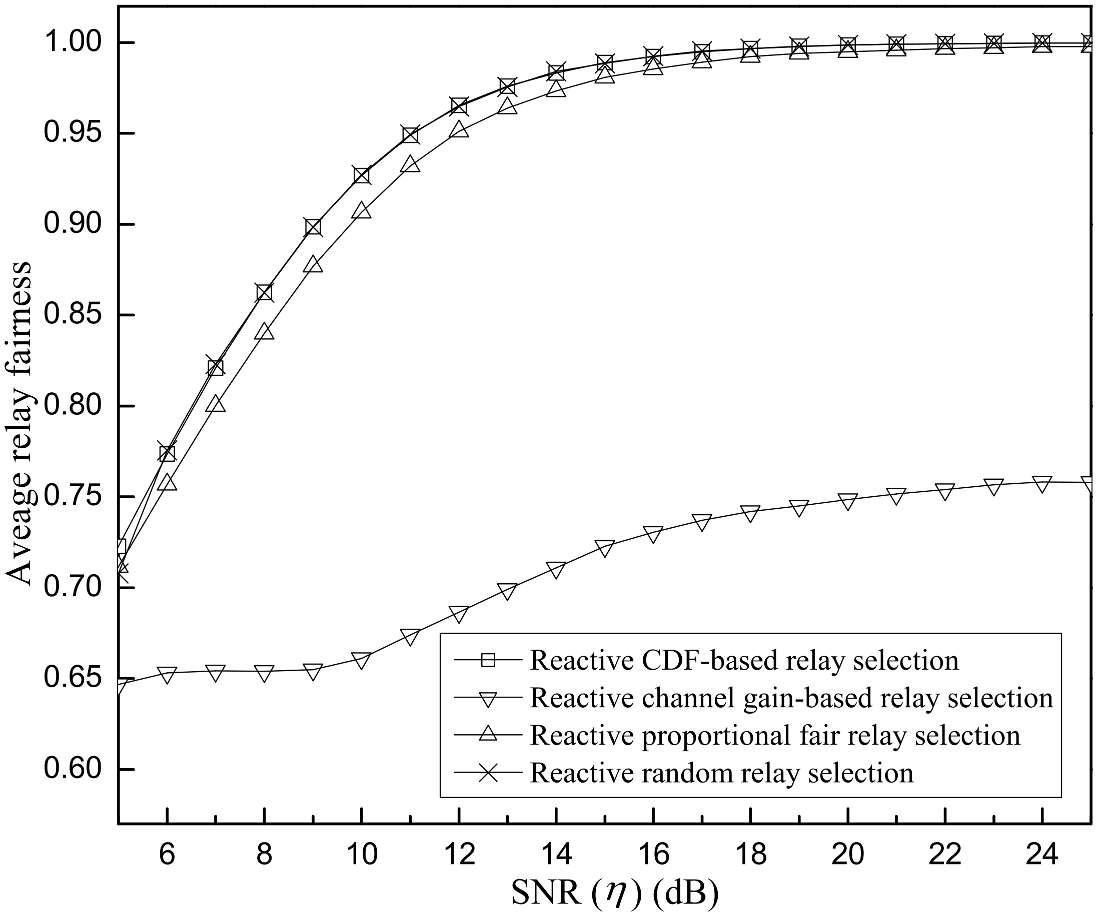


(b) Various values of m_{S,r_k} and $m_{r_k,D}$

Figure 2.4. Average relay fairness of reactive CDF-based relay selection scheme.



(a) $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{0.5, 1.0, 2.0\}$



(b) $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{1.0, 2.0, 3.0\}$

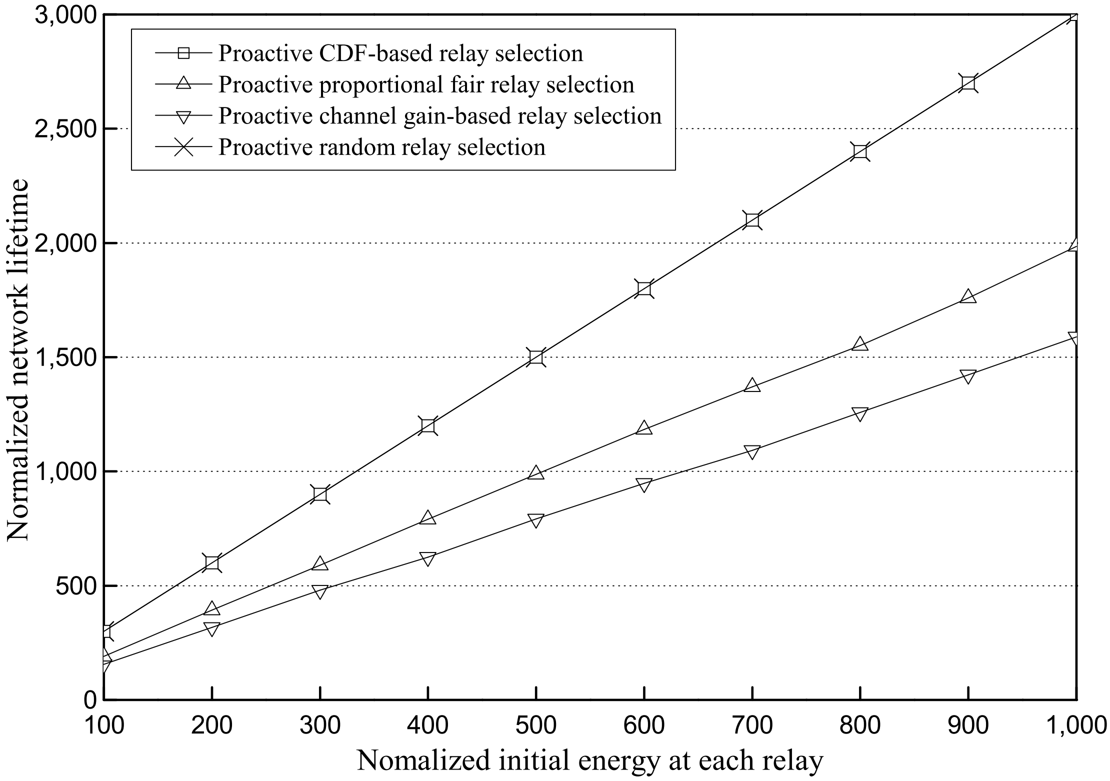
Figure 2.5. Average relay fairness of various reactive relay selection schemes.

2.4.2 Network Lifetime

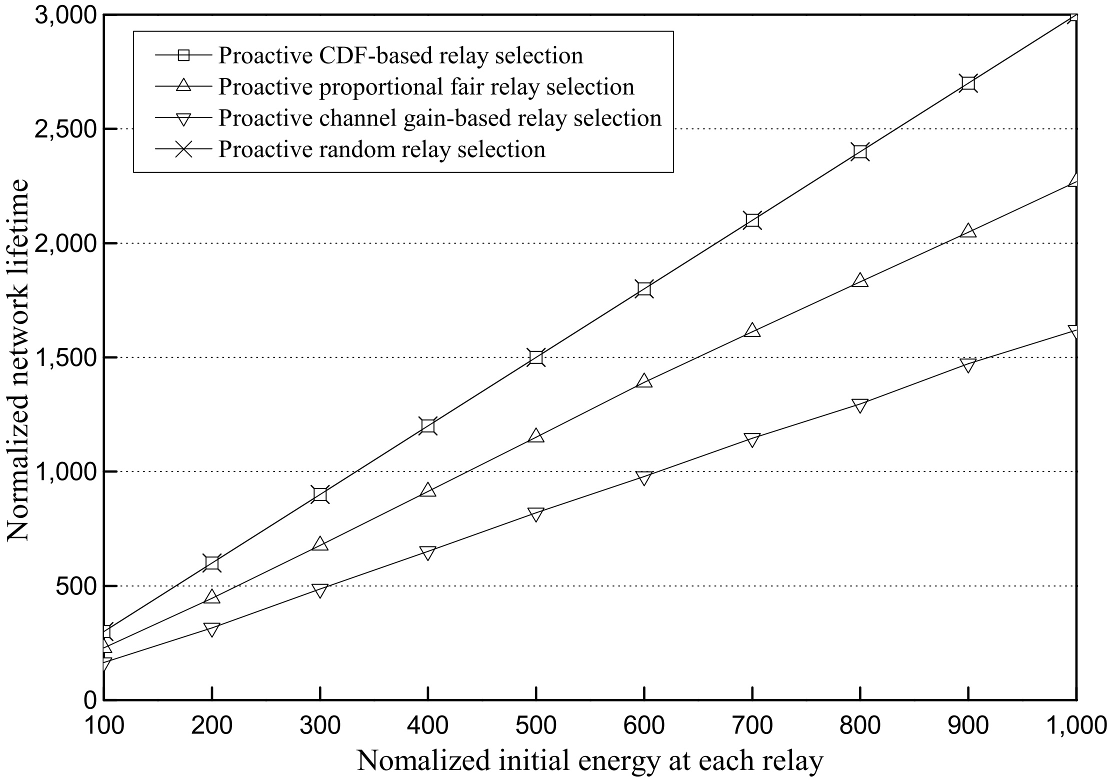
To provide insights into the impact of the average relay fairness on network lifetime, we will show network lifetime performances. Network lifetime is commonly defined as the time duration in which all nodes in the network remain active [90]-[94]. Suppose that the number of relays is three and the SNR is 20 dB. Suppose that $\{\Omega_{S,r_i}\}_{i=1}^{3} = \{\Omega_{r_i,D}\}_{i=1}^{3} = \{(1.2)^{-3}, (1.1)^{-3}, (1.0)^{-3}\}$.

Fig. 2.6 shows the network lifetime of various proactive relay selection schemes with various fading severity parameters. In figures, normalized initial energy at each relay means the initial energy when we assume that energy consumption for transmitting one packet is 1. Normalized network lifetime means the number of transmitted packet until one relay becomes inactive when we assume that the duration for transmitting one packet is 1. It is shown that proactive CDF-based relay selection scheme achieves higher network lifetime than other schemes except proactive random relay selection scheme. The reason that proactive CDF-based relay selection scheme and proactive random relay selection scheme achieve network lifetime is that they achieve same average relay fairness.

Fig. 2.7 shows the network lifetime of various reactive relay selection schemes with various fading severity parameters. It is shown that reactive CDF-based relay selection scheme achieves higher network lifetime than other schemes except reactive random relay selection scheme. Difference between the reactive CDF-based relay selection scheme and reactive proportional fair relay selection for $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{0.5, 1.0, 2.0\}$ is larger than that for $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{1.0, 2.0, 3.0\}$.

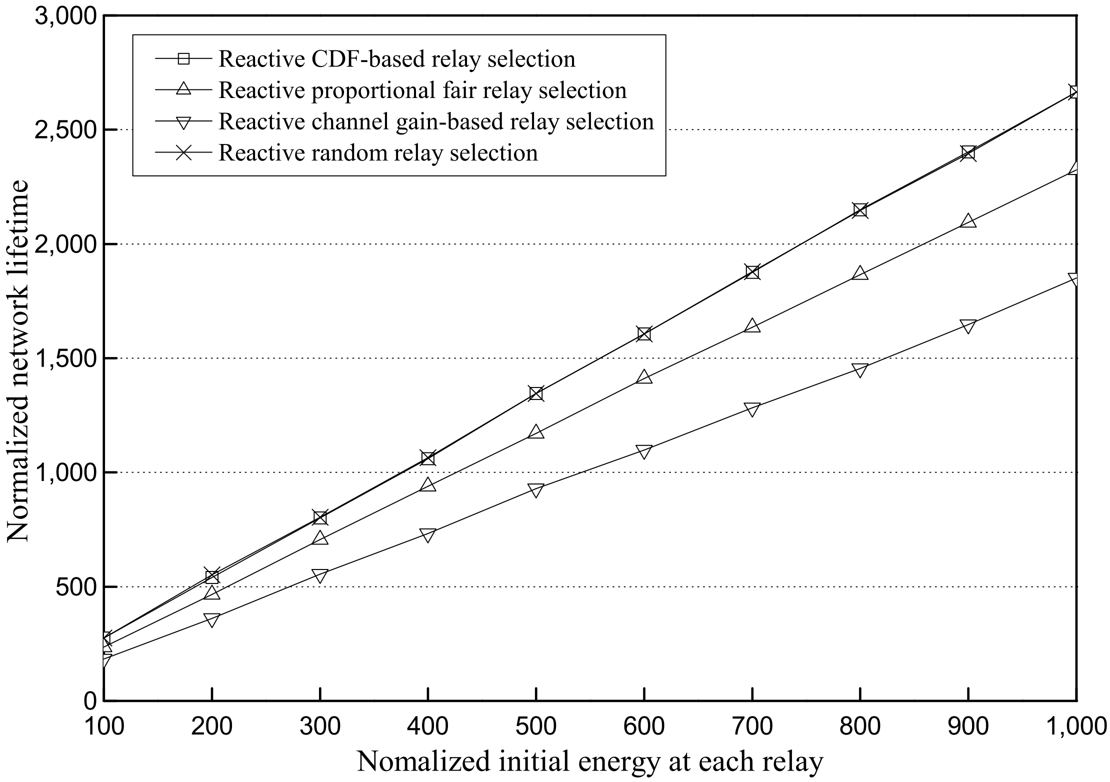


(a) $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{0.5, 1.0, 2.0\}$

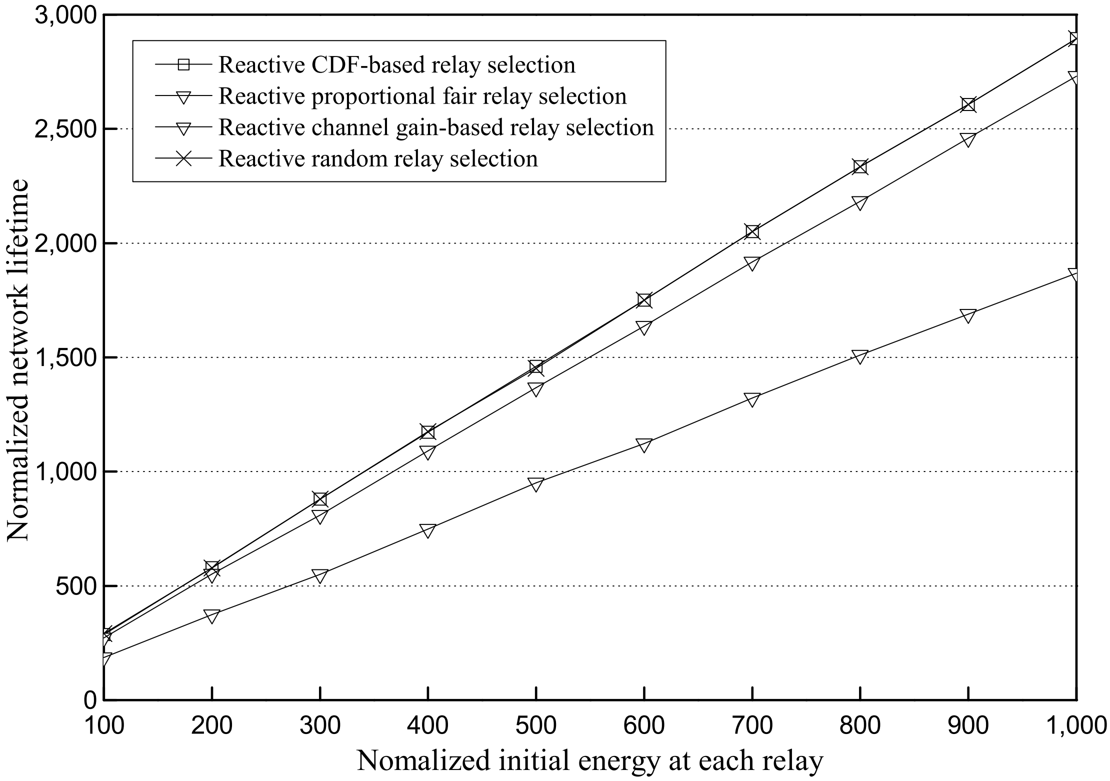


(b) $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{1.0, 2.0, 3.0\}$

Figure 2.6. Network lifetime of various proactive relay selection schemes.



(a) $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{0.5, 1.0, 2.0\}$



(b) $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{1.0, 2.0, 3.0\}$

Figure 2.7. Network lifetime of various reactive relay selection schemes.

2.4.3 Outage Probability

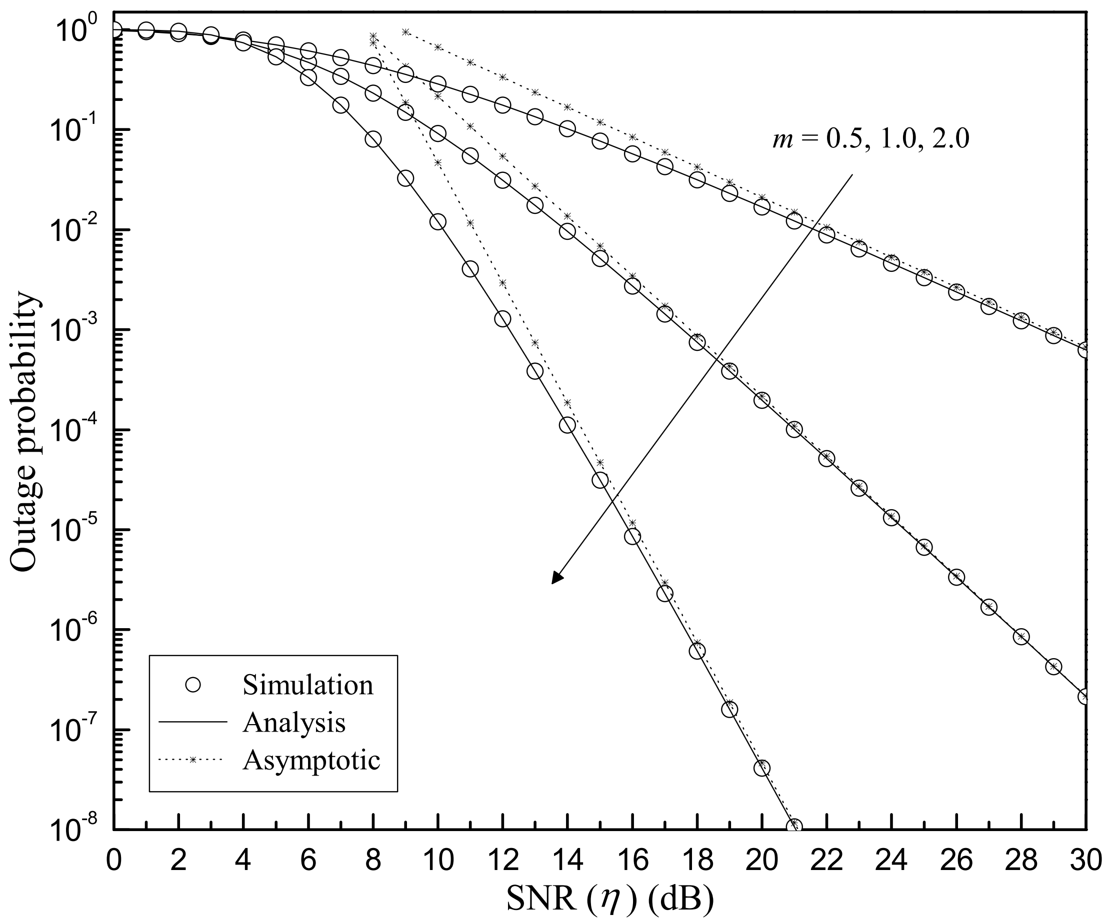
Fig. 2.8 shows outage probability of a network using the proactive CDF-based relay selection scheme with various fading severity parameters for $K = 3, 5$, respectively. In Fig. 2.8(a) and Fig. 2.8(b), we will use the notation $m_{S,r_k} = m_{r_k,D} = m$ to verify the effect of parameter $m_{i,j}$ on diversity order. Fig. 2.8(a) and Fig. 2.8(b) show that the outage probability analysis of the proactive CDF-based relay selection scheme perfectly matches the simulation results. The asymptotic analysis of the proactive CDF-based relay selection scheme is close to the simulation results at high SNR region. It is shown that as the value of parameter and the number of relays increase, the proactive CDF-based relay selection scheme achieves larger diversity order. Fig. 2.8(c) shows outage probability of a network with various fading severity parameters for $K = 3$. It is shown that the proactive CDF-based relay selection scheme in the case $\mathbf{m}_{S,r_k} = \{0.5, 0.5, 0.5\}$ and $\mathbf{m}_{r_k,D} = \{0.5, 0.5, 0.5\}$ achieves same diversity order as the case $\mathbf{m}_{S,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{r_k,D} = \{0.5, 1, 2\}$, the case $\mathbf{m}_{S,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{r_k,D} = \{2, 1, 0.5\}$, and the case $\mathbf{m}_{S,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{r_k,D} = \{1, 2, 3\}$.

Fig. 2.9 shows outage probabilities of various proactive relay selection schemes. It is shown that proactive channel gain-based relay selection scheme achieves lower outage probability than other relay selection schemes. It is shown that the proactive relay selection schemes for $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{1.0, 2.0, 3.0\}$ achieve lower outage probabilities than those for $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{0.5, 1.0, 1.5\}$.

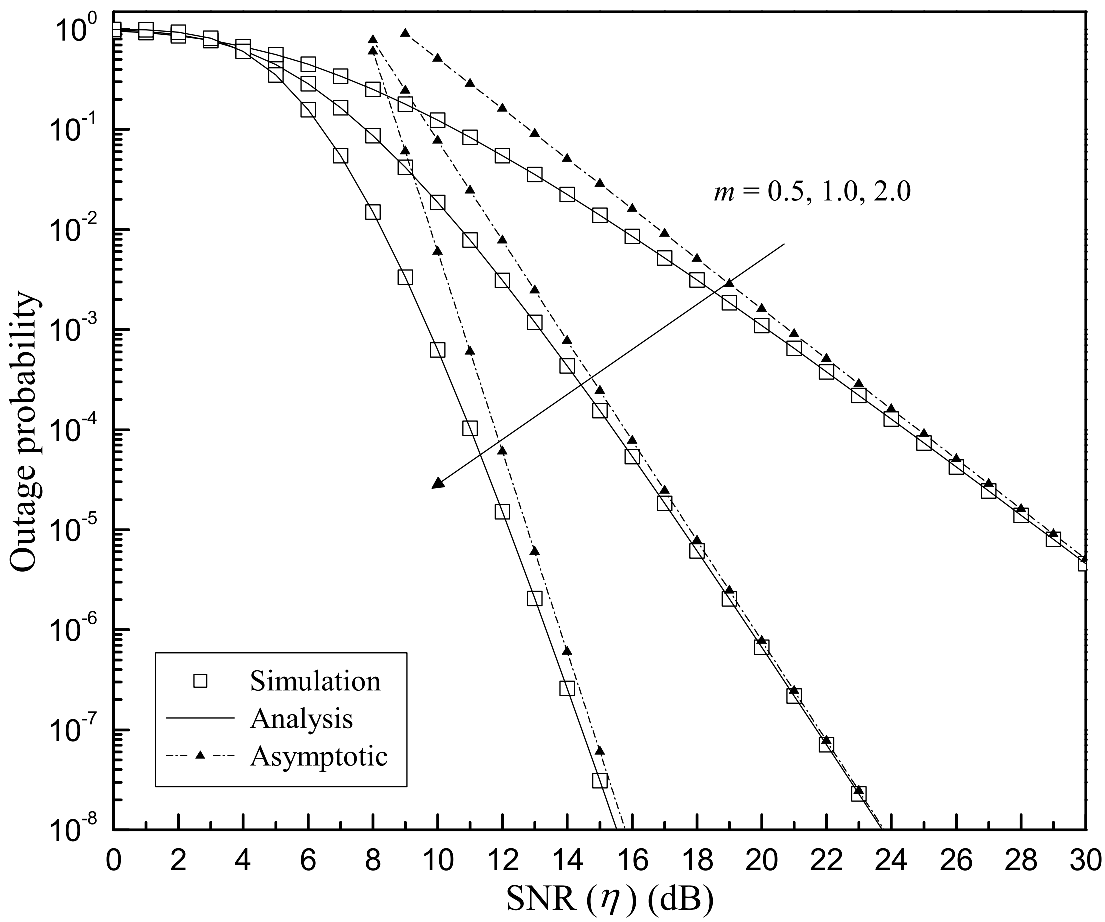
Fig. 2.10 shows outage probability of a network using the reactive CDF-based relay selection scheme with various fading severity parameters for $K = 3, 5$, respectively.

In Fig. 2.10(a) and Fig. 2.10(b), we will use the notation $m_{S,r_k} = m_{r_k,D} = m$ to verify the effect of parameter $m_{i,j}$ on diversity order. Fig. 2.10(a) and Fig. 2.10(b) show that the outage probability analysis of the reactive CDF-based relay selection scheme perfectly matches the simulation results. The asymptotic analysis of the reactive CDF-based relay selection scheme is close to the simulation results at high SNR region. It is shown that as the value of parameter and the number of relays increase, the reactive CDF-based relay selection scheme achieves larger diversity order. Fig. 2.10(c) shows outage probability of a network using the reactive CDF-based relay selection scheme with various fading severity parameters for $K = 3$. It is shown that the reactive CDF-based relay selection scheme in the case $\mathbf{m}_{S,r_k} = \{0.5, 0.5, 0.5\}$ and $\mathbf{m}_{r_k,D} = \{0.5, 0.5, 0.5\}$ achieves same diversity order as the case $\mathbf{m}_{S,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{r_k,D} = \{0.5, 1, 2\}$ and the case $\mathbf{m}_{S,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{r_k,D} = \{2, 1, 0.5\}$. Also, it is shown that the reactive CDF-based relay selection scheme in the case $\mathbf{m}_{S,r_k} = \{1, 1, 1\}$ and $\mathbf{m}_{r_k,D} = \{1, 1, 1\}$ achieves same diversity order as the case $\mathbf{m}_{S,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{r_k,D} = \{1, 2, 3\}$.

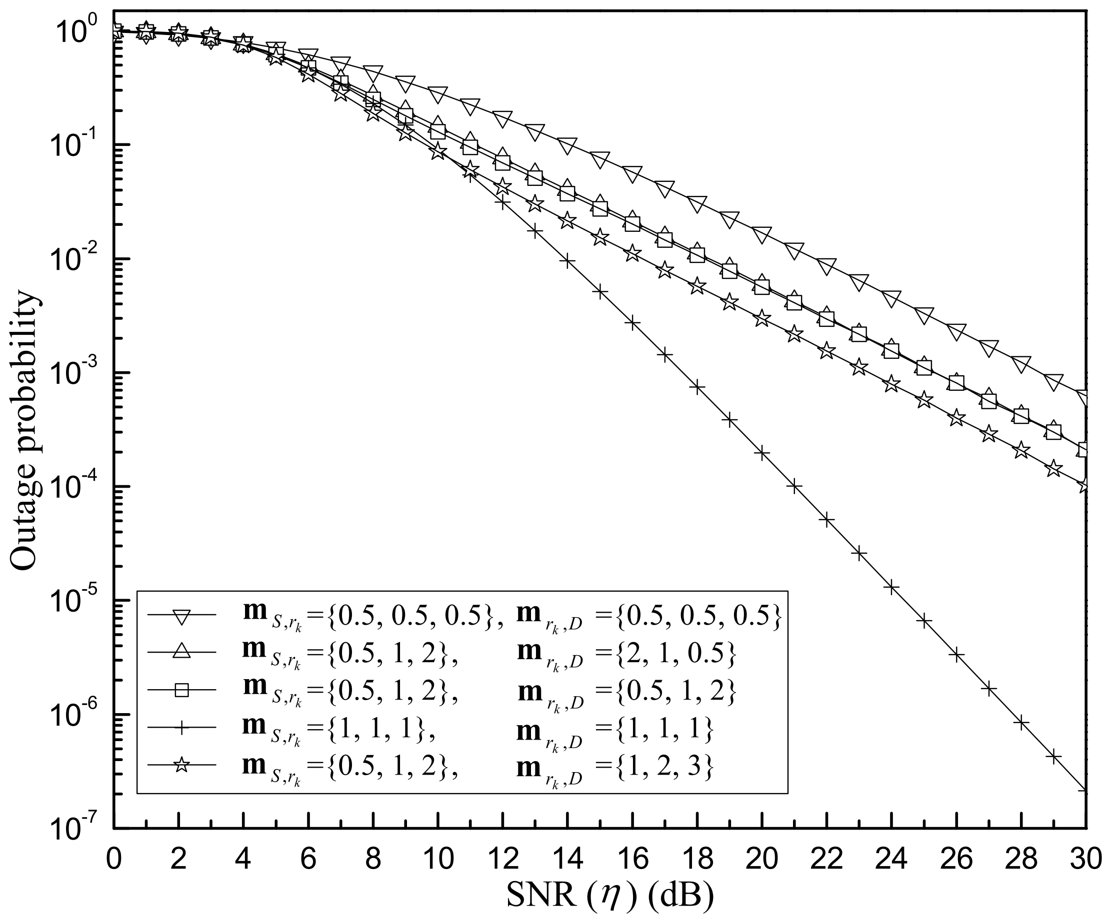
Fig. 2.11 shows outage probabilities of various reactive relay selection schemes. It is shown that reactive channel gain-based relay selection scheme achieves lower outage probability than other relay selection schemes. It is shown that the proactive relay selection schemes for $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{1.0, 2.0, 3.0\}$ achieve lower outage probabilities than those for $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{0.5, 1.0, 1.5\}$.



(a) $m_{S,r_k} = m_{r_k,D} = m$, $K = 3$

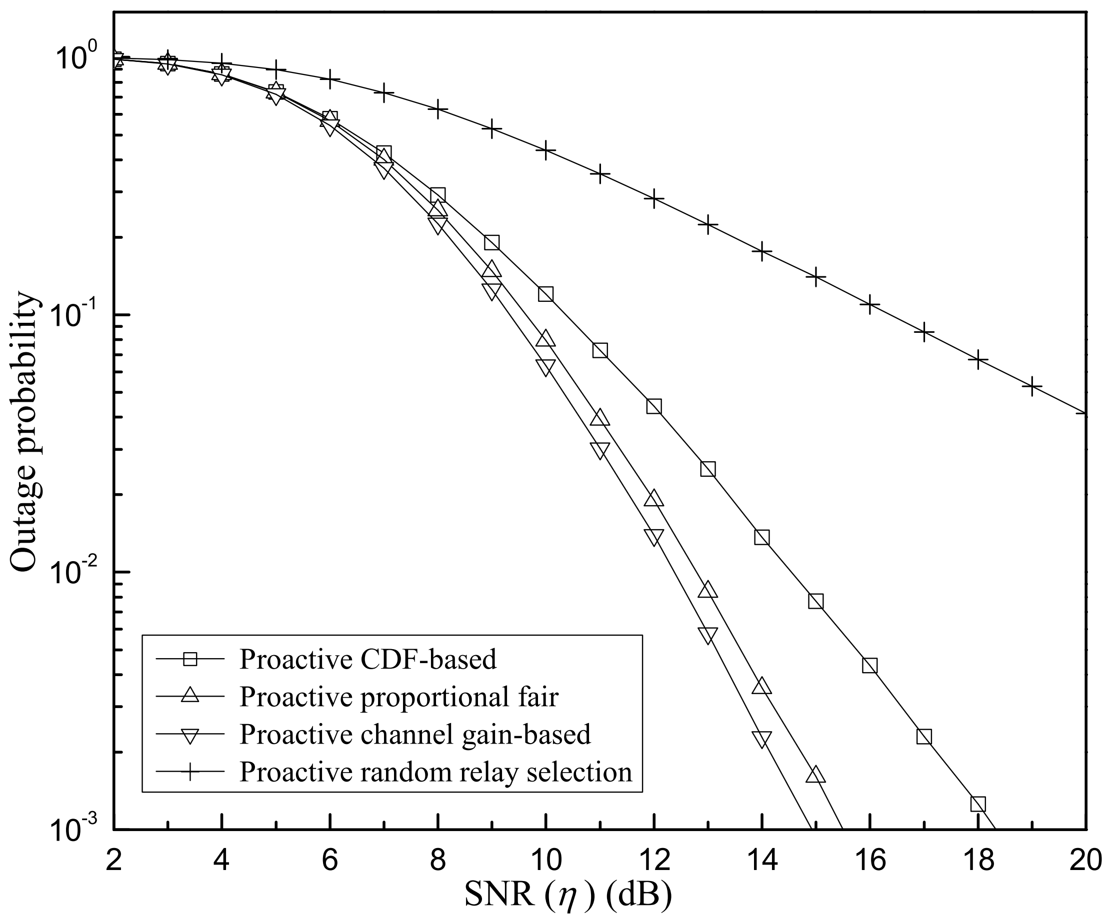


(b) $m_{S,r_k} = m_{r_k,D} = m$, $K = 5$

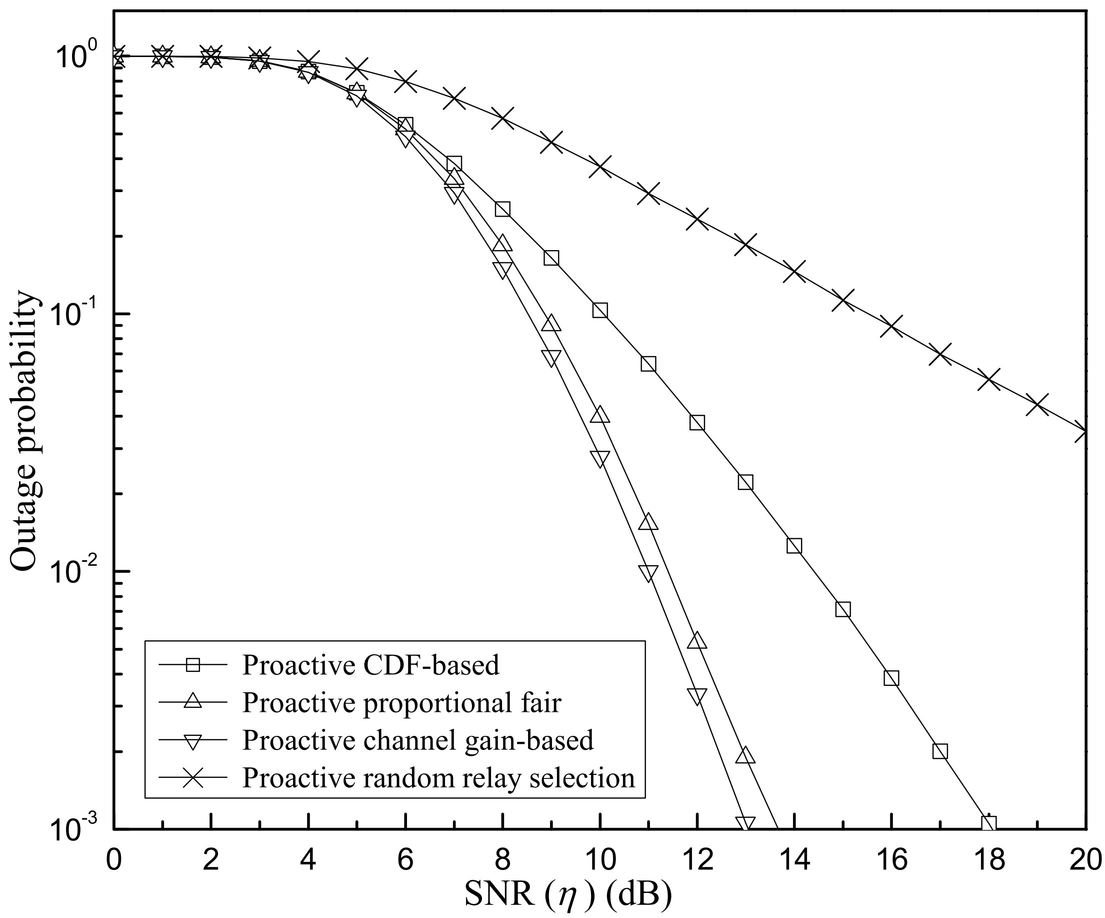


(c) Various values of m_{S,r_k} and $m_{r_k,D}$, $K = 3$

Figure 2.8. Outage probability of proactive CDF-based relay selection scheme.

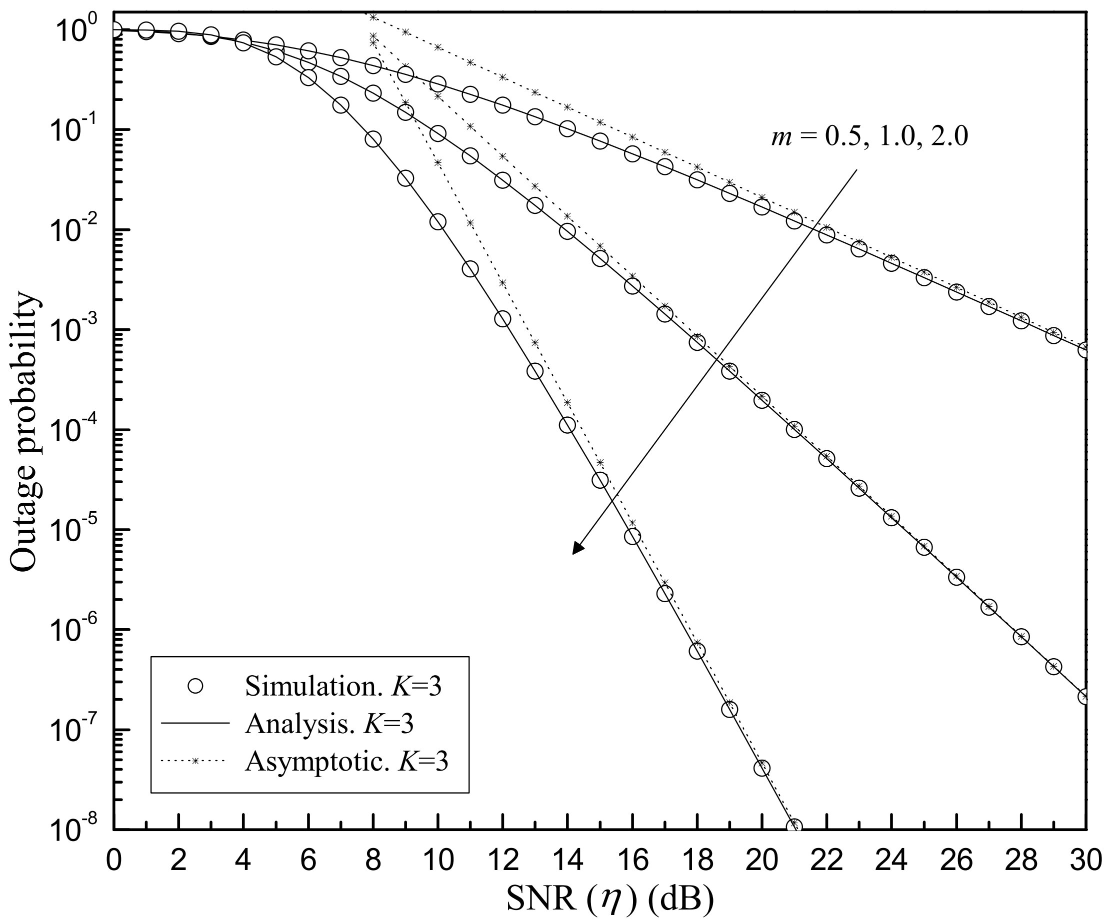


(a) $K = 3$, $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{0.5, 1.0, 1.5\}$

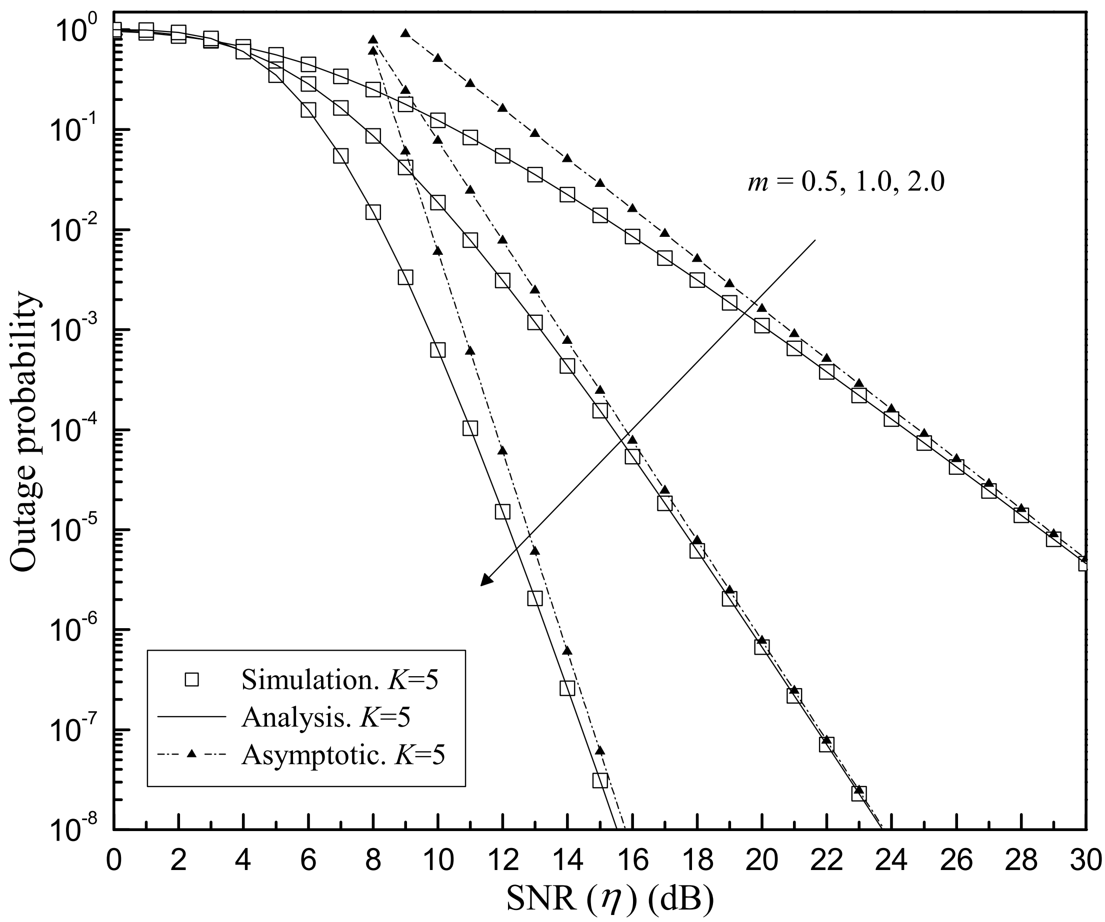


(b) $K = 3$, $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{1.0, 2.0, 3.0\}$

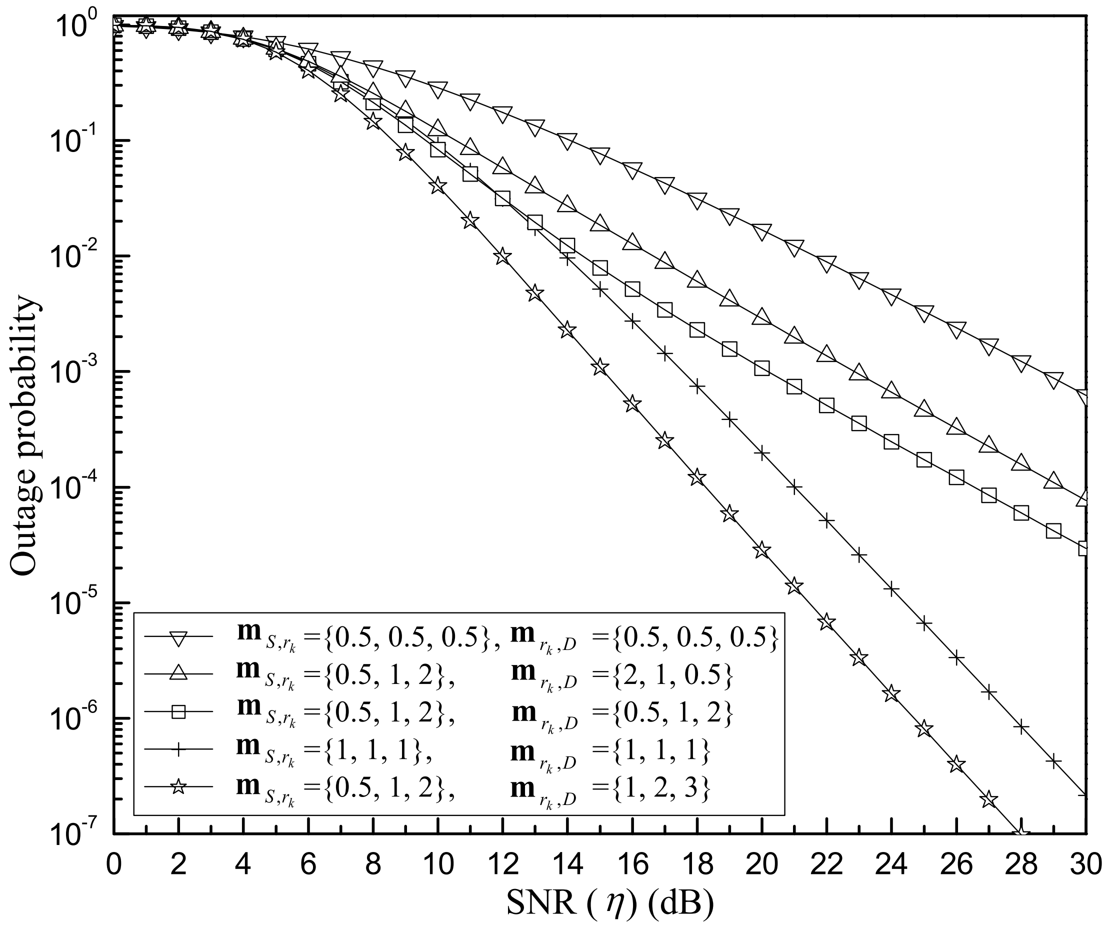
Figure 2.9. Outage probability of various proactive relay selection schemes.



(a) $K = 3$, $m_{S,r_k} = m_{r_k,D} = m$

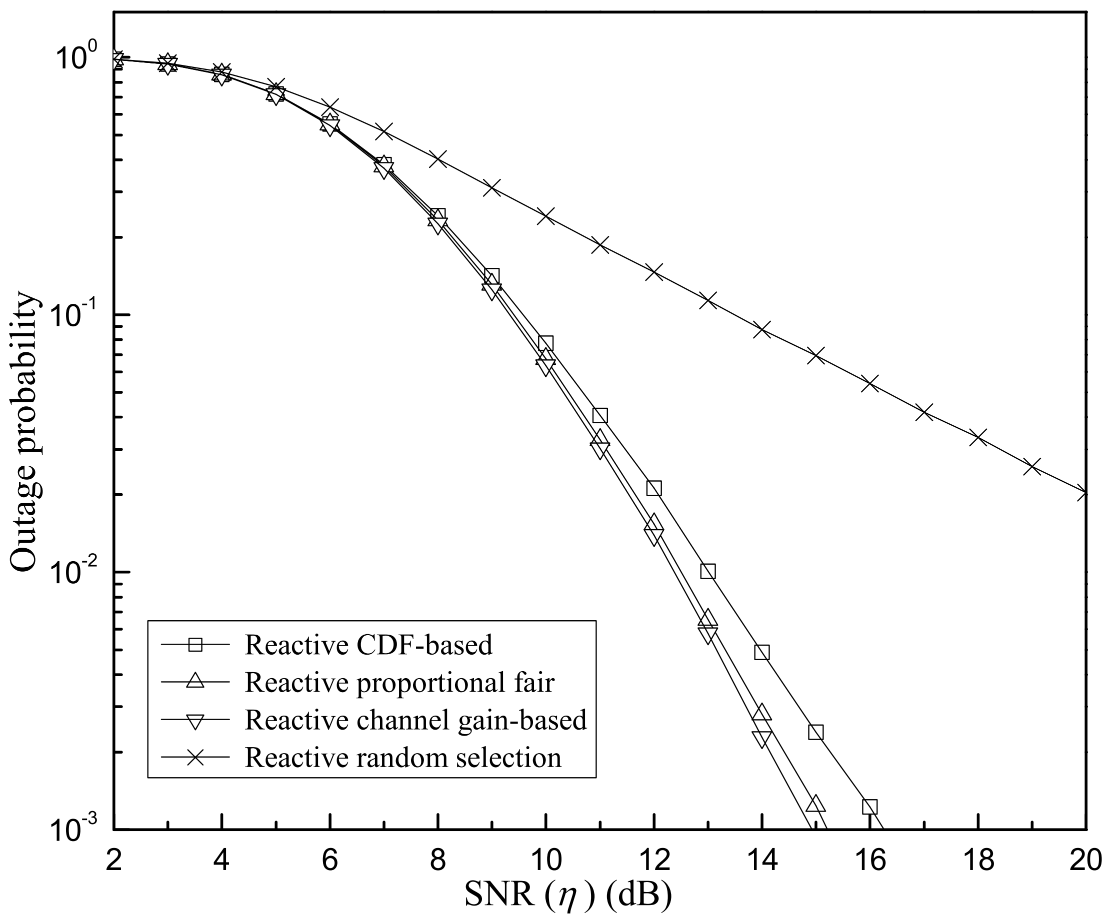


(b) $K = 5$, $m_{S,r_k} = m_{r_k,D} = m$

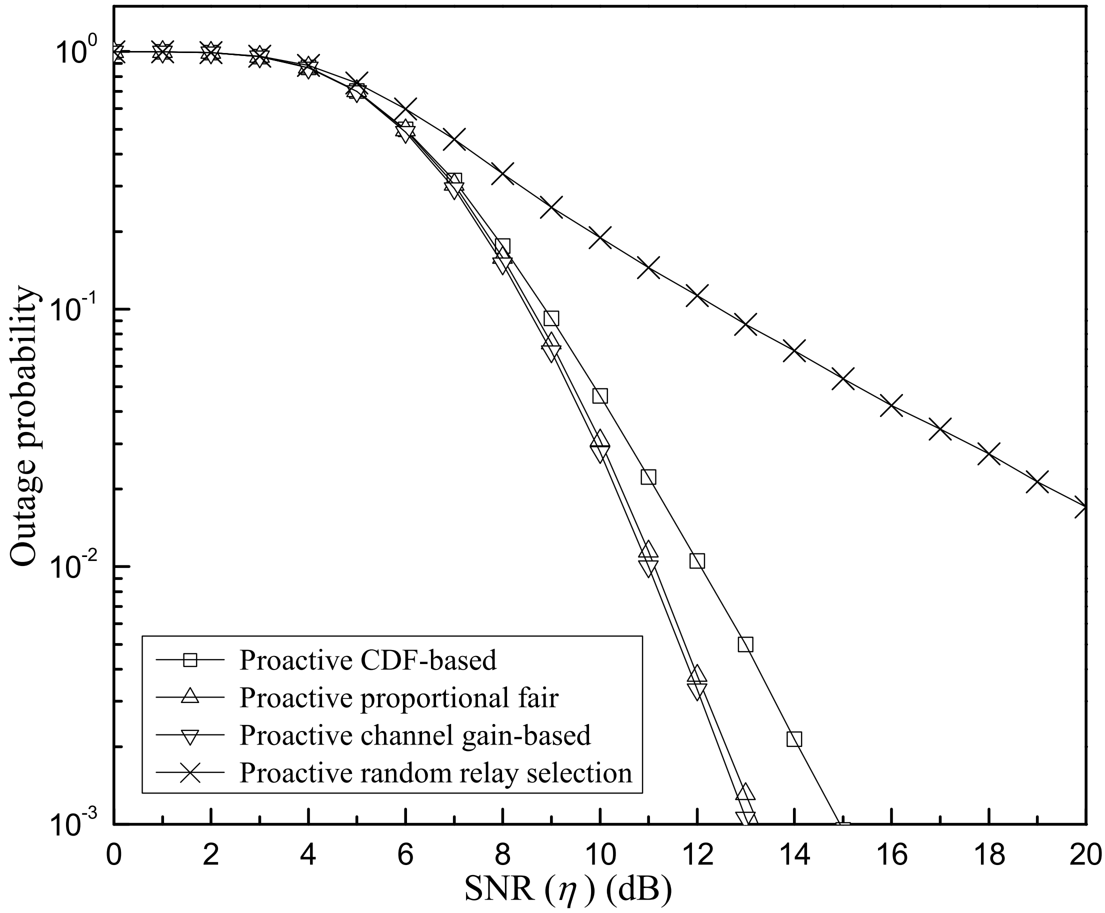


(c) $K = 3$, various values of m_{S,r_k} and $m_{r_k,D}$

Figure 2.10. Outage probability of reactive CDF-based relay selection scheme.



(a) $K = 3$, $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{0.5, 1.0, 1.5\}$



(b) $K = 3$, $\mathbf{m}_{S,r_k} = \mathbf{m}_{r_k,D} = \{1.0, 2.0, 3.0\}$

Figure 2.11. Outage probability of various reactive relay selection schemes.

2.5 Summary

In this chapter, we investigate the proactive and the reactive relay selection schemes based on CDFs of SNRs for one-way relay networks over Nakagami-m fading channels. For both the proactive and the reactive relay selection schemes, average relay fairness is analyzed by deriving relay selection probability to verify strictness of fairness for potential relays. For the proactive CDF-based relay selection scheme, diversity order is analyzed by deriving the integral and asymptotic expressions for outage probability. For the reactive CDF-based relay selection scheme, diversity order is obtained by deriving the exact and asymptotic expressions for outage probability. Analytical results are verified by Monte Carlo simulations. Numerical results show that the analytical results of average relay fairness and outage probability match the simulation results of them well. It is shown that the proactive CDF-based relay selection scheme guarantees strict fairness among relays regardless of the SNR. Whereas, reactive CDF-based relay selection scheme guarantees strict fairness among relay at high SNR region. To provide insights into the impact of the average relay fairness on network lifetime, we show network lifetime performances. It is shown that proactive CDF-based relay selection scheme achieves higher network lifetime than other schemes except proactive random relay selection scheme. Also, it is shown that reactive CDF-based relay selection scheme achieves higher network lifetime than other schemes except reactive random relay selection scheme. With respect to outage probability, it is shown that diversity order depends on the number of relays and fading severity parameters.

Chapter 3

Relay Selection Based on CDFs of SNRs for Two-Way Relay Networks

The drawback of one-way relaying is a loss in spectral efficiency due to half-duplex signaling. Two-way relaying is proposed to overcome low spectral efficiency of one-way relaying, and provide more spectral efficiency than one-way relaying [29]-[31]. Two-way relaying where two users communicate with each other via intermediate relays is an efficient way to avoid wireless impairments, obtain spatial diversity, and improve the throughput of a network [95]-[107].

In two-way relay networks (TWRNs) with multiple relays, relay selection is widely used and analyzed since it is simple to implement practically and does not need strict time synchronization among relays [98], [101]. Most of previous works use max-min signal-to-noise ratio (SNR)-based relay selection for TWRNs in which all channels experience same fading [98]-[107]. To the best of our knowledge, the relay selection adopting CDF-based scheduling for two-way relay networks where channels experience different fading has not been studied yet.

We propose the proactive and the reactive relay selection schemes based on CDFs of SNRs for two-way relay networks over Nakagami-m fading channels. For both the proactive and the reactive relay selection schemes, we analyze average relay fairness by deriving relay selection probability. For the proactive relay selection scheme, we derive the integral and asymptotic expressions for outage probability. Also, for the reactive relay selection scheme, we derive the asymptotic expressions for outage probability. Analytical results are verified by Monte Carlo simulations. The average relay fairness and the outage probability are compared with various relay selection schemes.

3.1 System Model

Consider a two-way relay network with two users, A and B, and the set of K potential relays, $\mathcal{R} = \{r_1, r_2, \cdots, r_K\}$. Suppose that each node has a single antenna and all nodes do not transmit and receive signals simultaneously. We adopt decode-and-forward (DF) protocol for relaying. Assume that the direct path between users A and B is not available. Assume that the channel coefficient from node i to node j, $h_{i,j}$, $i, j \in \{A, B, r_1, r_2, \cdots, r_K\}$, follows Nakagami distribution with the fading severity parameter $m_{i,j}$ and average fading power $\Omega_{i,j}$. Assume that all channels are reciprocal and have an additive white Gaussian noise (AWGN) with zero mean and variance N_0.

The received SNR at node j by transmission of node i is given by $Z_{i,j} = |h_{i,j}|^2 P_i / N_0$ where P_i is the transmit power of node i. Assume $P_A = P_B = P_{r_1} = \cdots = P_{r_K} = P$. The PDF and CDF of the received SNR $Z_{i,j}$ are given by

$$f_{Z_{i,j}}(z) = \frac{1}{\Gamma(m_{i,j})} \left(\frac{m_{i,j}}{\bar{z}_{i,j}} \right)^{m_{i,j}} z^{m_{i,j}-1} e^{\frac{m_{i,j} z}{\bar{z}_{i,j}}}, \quad z \geq 0, \tag{3.1}$$

and

$$F_{Z_{i,j}}(z) = \frac{\gamma\left(m_{i,j}, \frac{m_{i,j} z}{\bar{z}_{i,j}}\right)}{\Gamma(m_{i,j})}, \quad z \geq 0, \tag{3.2}$$

respectively, where $\bar{z}_{i,j} = \Omega_{i,j} P / N_0$. We consider a block fading model where all channel gains remain constant during the whole data transmission and change independently across different ones [85], [86]. Assume that users A and B transmit pilot signals and each relay obtains the distributions of the received SNRs by collecting channel state information from the pilot signals [73].

We consider both proactive and reactive relay selection depending on whether a relay is selected before or after users A and B broadcast. If the relay is chosen before the users transmission, it is called as *proactive relay selection.* On the other hand, if the relay is chosen after the users transmission, it is called as *reactive relay selection.*

3.1.1 Proactive CDF-based Relay Selection

When the received SNRs at users A and B from relay r_k have the values of Z_{A,r_k} and Z_{B,r_k}, respectively, and the received SNRs at relay r_k from users A and B have the values of z_{A,r_k} and z_{B,r_k}, respectively, a relay is selected based on its received SNRs such that

$$r^* = \arg\max_{r_k \in \mathcal{R}} \{\min\{F_{Z_{A,r_k}}(z_{A,r_k}),\ F_{Z_{B,r_k}}(z_{B,r_k})\}\}. \tag{3.3}$$

After a relay is selected, other relays except the selected relay remain silent and users A and B exchange information via a relay in three phases. In the first phase, user A broadcasts signal x_A to all potential relays. In the second phase, user B broadcasts signal x_B to all potential relays. The received signals at the relay r^* in the first phase and the second phase are given by

$$y_{r^*}^{(1)} = h_{A,r^*} x_A + n_{r^*}^{(1)}, \tag{3.4}$$

and

$$y_{r^*}^{(2)} = h_{B,r^*} x_B + n_{r^*}^{(2)}, \tag{3.5}$$

respectively, where $n_{r^*}^{(1)}$ and $n_{r^*}^{(2)}$ are AWGNs at the relay r^* in the first phase and the second phase, respectively.

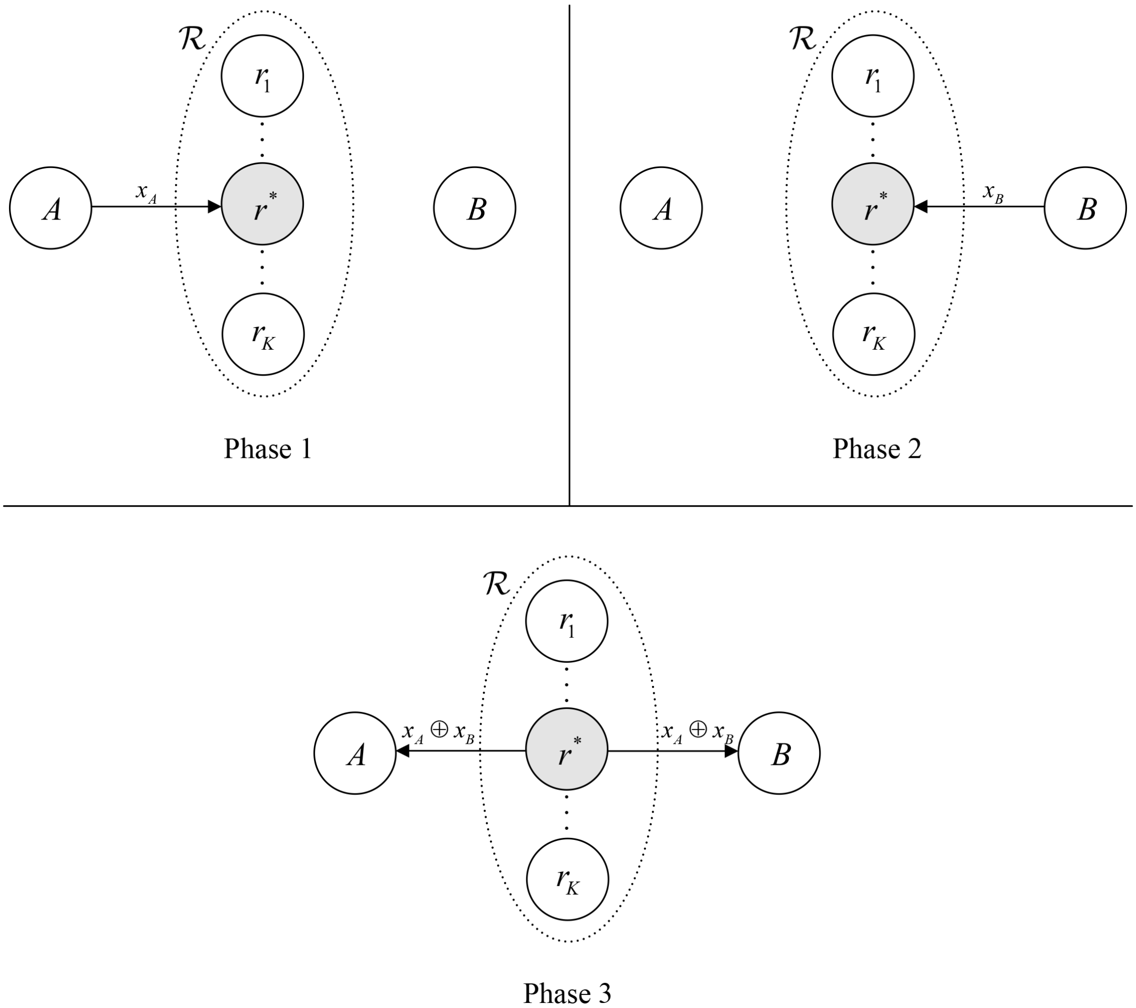
In the third phase, the selected relay r^* transmits the signal $x_{A,B} = x_A \oplus x_B$ to both users A and B, where $\oplus$ is an XOR operation. The received signals at users A and B are given by

$$y_A = h_{r^*,A} x_{A,B} + n_A^{(3)}, \tag{3.6}$$

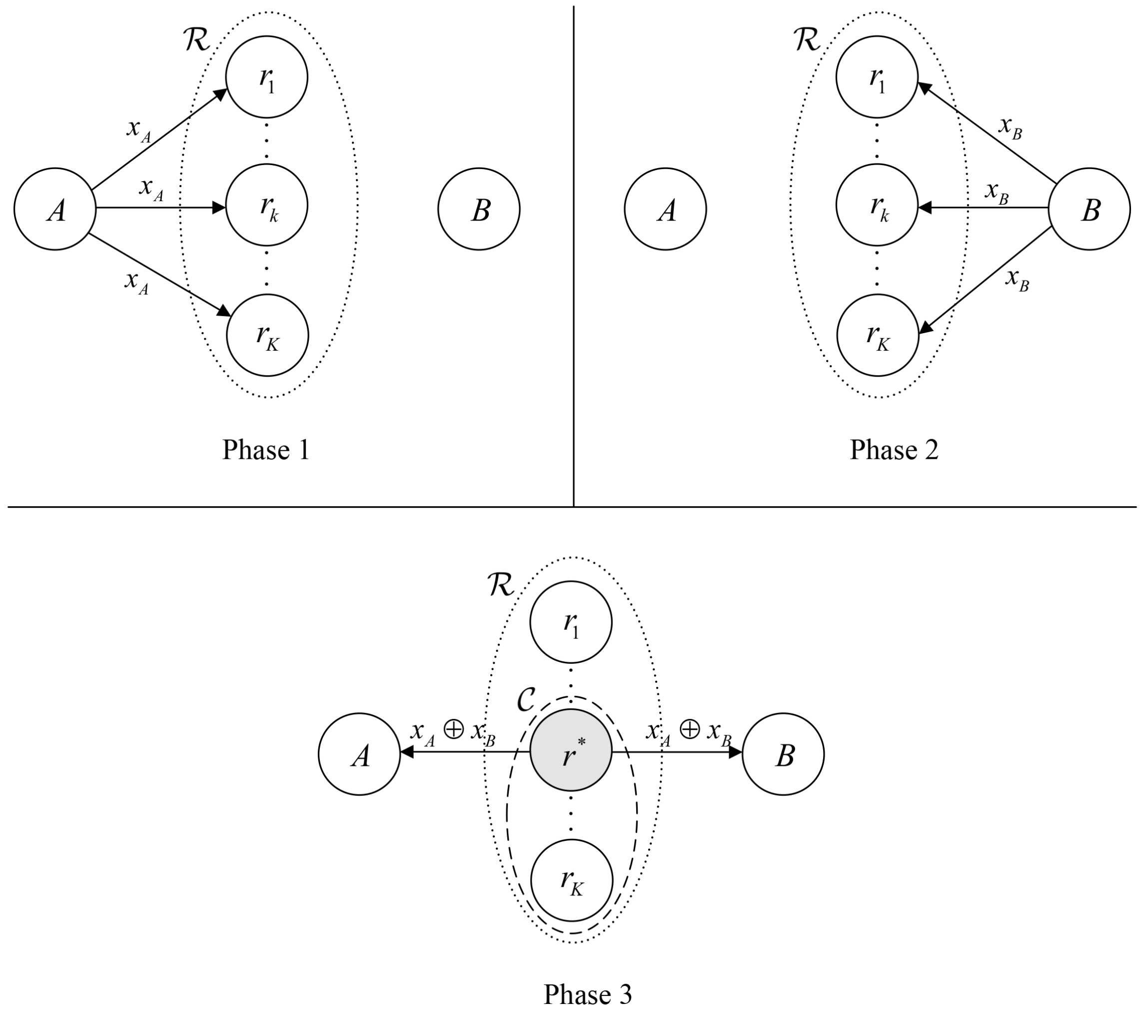
and

$$y_B = h_{r^*,B} x_{A,B} + n_B^{(3)}, \tag{3.7}$$

respectively, where $n_A^{(3)}$ and $n_B^{(3)}$ are AWGNs at users A and B in the third phase, respectively.



(a) Proactive relay selection



(b) Reactive relay selection

Figure 3.1. Proactive and reactive relay selection for two-way relay networks where users A and B exchange information with each other by the help of K relays. The shaded relay indicates the selected relay.

3.1.2 Reactive CDF-based Relay Selection

Unlike the proactive relay selection, a relay is selected after users A and B broadcast. Users A and B exchange data via a relay in three phases. In the first phase, user A broadcasts signal x_A to all potential relays with rate R. In the second phase, user B broadcasts signal x_B to all potential relays with rate R. The received signals at r_k in the first and the second phases are given by

$$y_{r_k}^{(1)} = h_{A,r_k} x_A + n_{r_k}^{(1)}, \tag{3.8}$$

and

$$y_{r_k}^{(2)} = h_{B,r_k} x_B + n_{r_k}^{(2)}, \tag{3.9}$$

respectively, where $n_{r_k}^{(1)}$ and $n_{r_k}^{(2)}$ are AWGNs. Assume that if the received SNR at the relay is larger than SNR threshold $z_{th} = 2^{3R} - 1$ for the target rate R, the relay correctly decodes its received signal. This equation is derived from

$$R = \frac{1}{3} \log(1 + z_{th}) \tag{3.10}$$

where the pre-log factor 1/3 comes from the fact that the users exchange information over three phases.

Let $\mathcal{C}$ denote the set of relays which successfully decode the received signals from both users A and B. When the received SNRs at relay r_k from users A and B have the instantaneous values of $z_{r_k,A}$ and $z_{r_k,B}$, respectively, a relay in $\mathcal{C}$ is selected based on CDFs of its received SNRs such that

$$r^* = \arg\max_{r_k \in \mathcal{C}} \{\min\{F_{Z_{r_k,A}}(z_{r_k,A}),\ F_{Z_{r_k,B}}(z_{r_k,B})\}\}. \tag{3.11}$$

Relay selection can be done distributively by timer-based method [64] where the relays set timer whose duration is inversely proportional to a metric depending only on their CDFs. The relay with the shortest timer duration becomes the selected one and notifies others about its availability.

Finally, in the third phase, the selected relay r^* transmits the signal $x_{A,B} = x_A \oplus x_B$ to both users A and B. The received signals at users A and B are given by

$$y_A = h_{r^*,A} x_{A,B} + n_A^{(3)}, \tag{3.12}$$

and

$$y_B = h_{r^*,B} x_{A,B} + n_B^{(3)}, \tag{3.13}$$

respectively, where $n_A^{(3)}$ and $n_B^{(3)}$ are AWGNs. After receiving signals from the relay, users A and B decode the received signals and subtract their own data from the decoded signal.

3.2 Performance Analysis of Proactive CDF-Based Relay Selection

3.2.1 Average Relay Fairness Analysis

Define the random variables $U_{A,k} \triangleq F_{Z_{A,r_k}}(Z_{A,r_k})$ and $U_{B,k} \triangleq F_{Z_{B,r_k}}(Z_{B,r_k})$. Note that $U_{A,k}$ and $U_{B,k}$ are i.i.d. uniform random variables ranging from 0 to 1 [73], [88]. Define a random variable $U_k \triangleq \min\{U_{A,k}, U_{B,k}\}$. Then, the probability of selecting r_k

is given by

$$\begin{aligned}
\Pr(r^* = r_k) &= \int_0^1 \left(\prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(U_i < u) \right) f_{U_k}(u) du \\
&= \int_0^1 \left(\prod_{\substack{i=1 \\ i \neq k}}^{K} F_{U_i}(u) \right) f_{U_k}(u) du \\
&= \int_0^1 (F_{U_k}(u))^{K-1} f_{U_k}(u) du \\
&= \frac{1}{K}. \qquad (3.14)
\end{aligned}$$

From (2.10), (2.11), and (3.14), the average relay fairness $\mathcal{F}$ becomes 1, which means that the proactive relay selection scheme achieves strict fairness among relays regardless of the SNR.

3.2.2 Outage Probability Analysis

An outage occurs when the SNR at the selected relay from either user A or user B is smaller than SNR threshold z_{th} or the SNR at either user A or user B from the selected relay is smaller than SNR threshold z_{th}. Define a random variable $Z_{r_k} \triangleq \min\{Z_{A,r_k}, Z_{B,r_k}\}$. Then, the outage probability is given by

$$\begin{aligned}
P_{out} &= \sum_{k=1}^{K} \Pr(Z_{r_k} < z_{th}, r^* = r_k) \\
&= \sum_{k=1}^{K} \int_0^{z_{th}} \Pr(r^* = r_k \mid Z_{r_k} = z) f_{Z_{r_k}}(z) dz \\
&= \sum_{k=1}^{K} I_k \qquad (3.15)
\end{aligned}$$

where $f_{Z_{r_k}}(\cdot)$ is the PDF of Z_{r_k} and

$$I_k = \int_0^{z_{th}} \Pr(r^* = r_k \,|\, Z_{r_k} = z) f_{Z_{r_k}}(z) dz. \tag{3.16}$$

By using the law of total probability, the probability of selecting r_k given that $Z_{r_k} = z$ is given by

$$\begin{aligned}
\Pr(r^* = r_k \,|\, Z_{r_k} = z) \;=\; & \Pr(r^* = r_k,\, U_{A,k} < U_{B,k},\, Z_{A,r_k} < Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
& + \Pr(r^* = r_k,\, U_{A,k} \geq U_{B,k},\, Z_{A,r_k} < Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
& + \Pr(r^* = r_k,\, U_{A,k} \geq U_{B,k},\, Z_{A,r_k} \geq Z_{B,r_k} \,|\, Z_{B,r_k} = z) \\
& + \Pr(r^* = r_k,\, U_{A,k} < U_{B,k},\, Z_{A,r_k} \geq Z_{B,r_k} \,|\, Z_{B,r_k} = z).
\end{aligned} \tag{3.17}$$

The first term of the right-hand side of (3.17) can be written as

$$\begin{aligned}
& \Pr(r^* = r_k,\, U_{A,k} < U_{B,k},\, Z_{A,r_k} < Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
& \quad = \prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(U_k \geq \min\{U_{A,i}, U_{B,i}\},\, U_{A,k} < U_{B,k},\, Z_{A,r_k} < Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
& \quad = \int_{\mathcal{A}_5} \prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(F_{Z_{A,r_k}}(z) \geq \min\{U_{A,i}, U_{B,i}\} \,|\, Z_{A,r_k} = z, Z_{B,r_k} = s) f_{Z_{B,r_k}}(s) ds
\end{aligned} \tag{3.18}$$

where $\mathcal{A}_5 = \{s \,|\, F_{Z_{A,r_k}}(z) < F_{Z_{B,r_k}}(s),\ z < s\}$. The first term of the right-hand side of (3.18) can be rewritten as

$$
\begin{aligned}
&\Pr(r^* = r_k,\, U_{A,k} < U_{B,k},\ Z_{A,r_k} < Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
&= \int_{\max\{z,\ a\}}^{\infty} \prod_{\substack{i=1 \\ i \neq k}}^{K} (1 - \Pr(F_{Z_{A,r_k}}(z) < \min\{U_{A,i}, U_{B,i}\} |\, Z_{A,r_k} = z, Z_{B,r_k} = s)) f_{Z_{B,r_k}}(s) ds \\
&= \int_{\max\{z,\ a\}}^{\infty} \prod_{\substack{i=1 \\ i \neq k}}^{K} (1 - \Pr(F_{Z_{A,r_k}}(z) < U_{A,i}, F_{Z_{A,r_k}}(z) < U_{B,i} \,|\, Z_{A,r_k} = z, Z_{B,r_k} = s)) \\
&\qquad\qquad \times f_{Z_{B,r_k}}(s) ds \\
&= \int_{\max\{z,\ a\}}^{\infty} \prod_{\substack{i=1 \\ i \neq k}}^{K} \{1 - (1 - F_{Z_{A,r_k}}(z))(1 - F_{Z_{A,r_k}}(z))\} f_{Z_{B,r_k}}(s) ds \\
&= \int_{\max\{z,\ a\}}^{\infty} \{1 - (1 - F_{Z_{A,r_k}}(z))^2\}^{K-1} f_{Z_{B,r_k}}(s) ds \\
&= \int_{\max\{z,\ a\}}^{\infty} \{2F_{Z_{A,r_k}}(z) - (F_{Z_{A,r_k}}(z))^2\}^{K-1} f_{Z_{B,r_k}}(s) ds
\end{aligned} \tag{3.19}
$$

where $a = F^{-1}_{Z_{B,r_k}}(F_{Z_{A,r_k}}(z))$. The second term of the right-hand side of (3.17) can be written as

$$
\begin{aligned}
&\Pr(r^* = r_k,\, U_{A,k} \geq U_{B,k},\ Z_{A,r_k} < Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
&\quad = \prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(U_k \geq \min\{U_{A,i}, U_{B,i}\},\, U_{A,k} \geq U_{B,k},\ Z_{A,r_k} < Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
&\quad = \int_{\mathcal{A}_6} \prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(F_{Z_{B,r_k}}(s) \geq \min\{U_{A,i}, U_{B,i}\} \,|\, Z_{A,r_k} = z, Z_{B,r_k} = s) f_{Z_{B,r_k}}(s) ds
\end{aligned} \tag{3.20}
$$

where $\mathcal{A}_6 = \{s \,|\, F_{Z_{A,r_k}}(z) \geq F_{Z_{B,r_k}}(s),\ z < s\}$. The second term of the right-hand side of (3.18) can be rewritten as

$$
\begin{aligned}
&\Pr(r^* = r_k,\, U_{A,k} \geq U_{B,k},\, Z_{A,r_k} < Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
&= \int_z^{\max\{z,\, a\}} \prod_{\substack{i=1 \\ i \neq k}}^{K} (1 - \Pr(F_{Z_{B,r_k}}(s) < \min\{U_{A,i}, U_{B,i}\} |\, Z_{A,r_k} = z, Z_{B,r_k} = s)) f_{Z_{B,r_k}}(s) ds \\
&= \int_z^{\max\{z,\, a\}} \prod_{\substack{i=1 \\ i \neq k}}^{K} (1 - \Pr(F_{Z_{B,r_k}}(s) < U_{A,i}, F_{Z_{B,r_k}}(s) < U_{B,i} \,|\, Z_{A,r_k} = z, Z_{B,r_k} = s)) \\
&\qquad\qquad \times f_{Z_{B,r_k}}(s) ds \\
&= \int_z^{\max\{z,\, a\}} \prod_{\substack{i=1 \\ i \neq k}}^{K} \{1 - (1 - F_{Z_{B,r_k}}(s))(1 - F_{Z_{B,r_k}}(s))\} f_{Z_{B,r_k}}(s) ds \\
&= \int_z^{\max\{z,\, a\}} \{1 - (1 - F_{Z_{B,r_k}}(s))(1 - F_{Z_{B,r_k}}(s))\}^{K-1} f_{Z_{B,r_k}}(s) ds \\
&= \int_z^{\max\{z,\, a\}} \{2F_{Z_{B,r_k}}(s) - (F_{Z_{B,r_k}}(s))^2\}^{K-1} f_{Z_{B,r_k}}(s) ds. \qquad (3.21)
\end{aligned}
$$

The third term of the right-hand side of (3.17) can be written as

$$
\begin{aligned}
&\Pr(r^* = r_k,\, U_{A,k} \geq U_{B,k},\, Z_{A,r_k} \geq Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
&\quad = \int_{\mathcal{A}_7} \prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(F_{Z_{B,r_k}}(z) \geq \min\{U_{A,i}, U_{B,i}\} \,|\, Z_{A,r_k} = s, Z_{B,r_k} = z) f_{Z_{A,r_k}}(s) ds \quad (3.22)
\end{aligned}
$$

where $\mathcal{A}_7 = \{s \,|\, F_{Z_{A,r_k}}(z) \geq F_{Z_{B,r_k}}(s),\ z \geq s\}$. Similarly, the third term of the right-hand side of (3.22) can be rewritten as

$$
\begin{aligned}
&\Pr(r^* = r_k,\, U_{A,k} \geq U_{B,k},\, Z_{A,r_k} \geq Z_{B,r_k} \,|\, Z_{A,r_k} = z) \\
&\quad = \int_{\max\{z,\, b\}}^{\infty} \{2F_{Z_{B,r_k}}(z) - (F_{Z_{B,r_k}}(z))^2\}^{K-1} f_{Z_{A,r_k}}(s) ds \qquad (3.23)
\end{aligned}
$$

where $b = F^{-1}_{Z_{A,r_k}}(F_{Z_{B,r_k}}(z))$. The fourth term of the right-hand side of (3.17) can be written as

$$\begin{aligned}
&\Pr(r^* = r_k,\, U_{A,k} < U_{B,k},\, Z_{A,r_k} \geq Z_{B,r_k} \,|\, Z_{B,r_k} = z) \\
&\quad = \int_{\mathcal{A}_8} \prod_{\substack{i=1 \\ i \neq k}}^{K} \Pr(F_{Z_{B,r_k}}(s) \geq \min\{U_{A,i}, U_{B,i}\} \,|\, Z_{A,r_k} = s, Z_{B,r_k} = z) f_{Z_{A,r_k}}(s) ds \quad (3.24)
\end{aligned}$$

where $\mathcal{A}_8 = \{s \,|\, F_{Z_{A,r_k}}(z) < F_{Z_{B,r_k}}(s),\ z \geq s\}$. Similarly, the fourth term of the right-hand side of (3.25) can be rewritten as

$$\begin{aligned}
&\Pr(r^* = r_k,\, U_{A,k} < U_{B,k},\, Z_{A,r_k} \geq Z_{B,r_k} \,|\, Z_{B,r_k} = z) \\
&\quad = \int_{z}^{\max\{z,\, b\}} \{2F_{Z_{A,r_k}}(s) - (F_{Z_{A,r_k}}(s))^2\}^{K-1} f_{Z_{A,r_k}}(s) ds. \quad (3.25)
\end{aligned}$$

From (3.17), (3.19), (3.21), (3.23), and (3.25), I_k is given by

$$\begin{aligned}
I_k = &\int_0^{z_{th}} \int_{\max\{z,\, a\}}^{\infty} \{2F_{Z_{A,r_k}}(z) - (F_{Z_{A,r_k}}(z))^2\}^{K-1} f_{Z_{B,r_k}}(s) f_{Z_{A,r_k}}(z) ds dz \\
&+ \int_0^{z_{th}} \int_{z}^{\max\{z,\, a\}} \{2F_{Z_{B,r_k}}(s) - (F_{Z_{B,r_k}}(s))^2\}^{K-1} f_{Z_{B,r_k}}(s) f_{Z_{A,r_k}}(z) ds dz \\
&+ \int_0^{z_{th}} \int_{\max\{z,\, b\}}^{\infty} \{2F_{Z_{B,r_k}}(z) - (F_{Z_{B,r_k}}(z))^2\}^{K-1} f_{Z_{A,r_k}}(s) f_{Z_{B,r_k}}(z) ds dz \\
&+ \int_0^{z_{th}} \int_{z}^{\max\{z,\, b\}} \{2F_{Z_{A,r_k}}(s) - (F_{Z_{A,r_k}}(s))^2\}^{K-1} f_{Z_{A,r_k}}(s) f_{Z_{B,r_k}}(z) ds dz. \quad (3.26)
\end{aligned}$$

In the case that $m_{A,r_k} \geq m_{B,r_k}$, the first term on the right-hand side of (3.26) is given by

$$\begin{aligned}
&\int_0^{z_{th}} \int_{\max\{z,\, a\}}^{\infty} \{2F_{Z_{A,r_k}}(z) - (F_{Z_{A,r_k}}(z))^2\}^{K-1} f_{Z_{B,r_k}}(s) f_{Z_{A,r_k}}(z) ds dz \\
&\quad = \int_0^{z_{th}} \{2F_{Z_{A,r_k}}(z) - (F_{Z_{A,r_k}}(z))^2\}^{K-1} f_{Z_{A,r_k}}(z)(1 - F_{Z_{r_k,B}}(z)) dz \\
&\quad = \int_0^{z_{th}} \{2F_{Z_{A,r_k}}(z) - (F_{Z_{A,r_k}}(z))^2\}^{K-1} f_{Z_{A,r_k}}(z) dz \\
&\qquad - \int_0^{z_{th}} \{2F_{Z_{A,r_k}}(z) - (F_{Z_{A,r_k}}(z))^2\}^{K-1} f_{Z_{A,r_k}}(z) F_{Z_{B,r_k}}(z) dz. \quad (3.27)
\end{aligned}$$

The second term on the right-hand side of (3.26) is zero since $\max\{z,\ a\} = z$. The third term on the right-hand side of (3.26) is given by

$$\begin{aligned}
&\int_0^{z_{th}} \int_{\max\{z,\ b\}}^{\infty} \{2F_{Z_{B,r_k}}(z) - (F_{Z_{B,r_k}}(z))^2\}^{K-1} f_{Z_{A,r_k}}(s) f_{Z_{B,r_k}}(z) ds dz \\
&\qquad = \int_0^{z_{th}} \{2F_{Z_{B,r_k}}(z) - (F_{Z_{B,r_k}}(z))^2\}^{K-1} f_{Z_{B,r_k}}(z)(1 - F_{Z_{B,r_k}}(z)) dz \\
&\qquad = \int_0^{F_{Z_{B,r_k}}(z_{th})} (2t - t^2)^{K-1}(1-t) dt \\
&\qquad = \frac{1}{2K}\{2F_{Z_{B,r_k}}(z_{th}) - (F_{Z_{B,r_k}}(z_{th}))^2\}^K. \qquad (3.28)
\end{aligned}$$

The last term on the right-hand side of (3.26) is given by

$$\begin{aligned}
&\int_0^{z_{th}} \int_z^{\max\{z,\ b\}} \{2F_{Z_{A,r_k}}(s) - (F_{Z_{A,r_k}}(s))^2\}^{K-1} f_{Z_{A,r_k}}(s) f_{Z_{B,r_k}}(z) ds dz \\
&\qquad = \int_0^{z_{th}} \int_z^{b} \{2F_{Z_{A,r_k}}(s) - (F_{Z_{A,r_k}}(s))^2\}^{K-1} f_{Z_{A,r_k}}(s) f_{Z_{B,r_k}}(z) ds dz \\
&\qquad = \int_0^{z_{th}} \int_{F_{Z_{A,r_k}}(z)}^{F_{Z_{B,r_k}}(z)} (2t - t^2)^{K-1} dt f_{Z_{B,r_k}}(z) dz. \qquad (3.29)
\end{aligned}$$

From (3.26), (3.27), (3.28), and (3.29), we obtain integral I_k as

$$\begin{aligned}
I_k = &\int_0^{F_{Z_{A,r_k}}(z_{th})} (2t - t^2)^{K-1} dt + \frac{1}{2K}\{2F_{Z_{B,r_k}}(z_{th}) - (F_{Z_{B,r_k}}(z_{th}))^2\}^K \\
&- \int_0^{z_{th}} \{2F_{Z_{A,r_k}}(z) - (F_{Z_{A,r_k}}(z))^2\}^{K-1} f_{Z_{A,r_k}}(z) F_{Z_{B,r_k}}(z) dz \\
&+ \int_0^{z_{th}} \int_{F_{Z_{A,r_k}}(z)}^{F_{Z_{B,r_k}}(z)} (2t - t^2)^{K-1} dt f_{Z_{B,r_k}}(z) dz. \qquad (3.30)
\end{aligned}$$

When $\bar{z}_{i,j}$ is high, the incomplete gamma function can be approximated as [61, eq. (8.354.1)]

$$\gamma\left(m_{i,j}, \frac{m_{i,j} z}{\bar{z}_{i,j}}\right) \approx \frac{1}{m_{i,j}} \left(\frac{m_{i,j} z}{\bar{z}_{i,j}}\right)^{m_{i,j}}. \qquad (3.31)$$

At high SNR region, the last term on the right-hand side of (3.30) goes to zero. Hence, I_k can be approximated as

$$I_k \approx \frac{1}{2K}\{2F_{Z_{A,r_k}}(z_{th}) - (F_{Z_{A,r_k}}(z_{th}))^2\}^K + \frac{1}{2K}\{2F_{Z_{B,r_k}}(z_{th}) - (F_{Z_{B,r_k}}(z_{th}))^2\}^K. \quad (3.32)$$

Similarly, in the case that $m_{S,r_k} < m_{r_k,D}$, I_k is approximated as

$$I_k \approx \frac{1}{2K}\{2F_{Z_{B,r_k}}(z_{th}) - (F_{Z_{B,r_k}}(z_{th}))^2\}^K + \frac{1}{2K}\{2F_{Z_{A,r_k}}(z_{th}) - (F_{Z_{A,r_k}}(z_{th}))^2\}^K. \quad (3.33)$$

Then, the outage probability is approximated as

$$\begin{aligned} P_{out} \approx \sum_{k=1}^{K} \Bigg[& \frac{1}{2K}\{2F_{Z_{A,r_k}}(z_{th}) - (F_{Z_{A,r_k}}(z_{th}))^2\}^K \\ & + \frac{1}{2K}\{2F_{Z_{B,r_k}}(z_{th}) - (F_{Z_{B,r_k}}(z_{th}))^2\}^K \Bigg]. \end{aligned} \quad (3.34)$$

From (3.31) and (3.34), the outage probability is approximated as

$$\begin{aligned} P_{out} \approx & \frac{1}{2K}\sum_{k=1}^{K}\Bigg[\Bigg\{\frac{1}{m_{A,r_k}\Gamma(m_{A,r_k})}\left(\frac{m_{A,r_k}z_{th}}{\bar{z}_{A,r_k}}\right)^{m_{A,r_k}}\left(2-\frac{1}{m_{A,r_k}\Gamma(m_{A,r_k})}\left(\frac{m_{A,r_k}z_{th}}{\bar{z}_{A,r_k}}\right)^{m_{A,r_k}}\right)\Bigg\}^K \\ & + \Bigg\{\frac{1}{m_{B,r_k}\Gamma(m_{B,r_k})}\left(\frac{m_{B,r_k}z_{th}}{\bar{z}_{B,r_k}}\right)^{m_{B,r_k}}\left(2-\frac{1}{m_{B,r_k}\Gamma(m_{B,r_k})}\left(\frac{m_{B,r_k}z_{th}}{\bar{z}_{B,r_k}}\right)^{m_{B,r_k}}\right)\Bigg\}^K\Bigg] \\ \approx & \frac{1}{2K}\sum_{k=1}^{K}\Bigg[\Bigg\{\frac{1}{m_{A,r_k}\Gamma(m_{A,r_k})}\left(\frac{m_{A,r_k}z_{th}}{\Omega_{A,r_k}\eta}\right)^{m_{A,r_k}}\left(\frac{2m_{A,r_k}\Gamma(m_{A,r_k})\Omega_{A,r_k}^{m_{A,r_k}}\eta^{m_{A,r_k}} - m_{A,r_k}^{m_{A,r_k}}z_{th}^{m_{A,r_k}}}{m_{A,r_k}\Gamma(m_{A,r_k})\Omega_{A,r_k}^{m_{A,r_k}}\eta^{m_{A,r_k}}}\right)\Bigg\}^K \\ & + \Bigg\{\frac{1}{m_{B,r_k}\Gamma(m_{B,r_k})}\left(\frac{m_{B,r_k}z_{th}}{\Omega_{B,r_k}\eta}\right)^{m_{B,r_k}}\left(\frac{2m_{B,r_k}\Gamma(m_{B,r_k})\Omega_{B,r_k}^{m_{B,r_k}}\eta^{m_{B,r_k}} - m_{B,r_k}^{m_{B,r_k}}z_{th}^{m_{B,r_k}}}{m_{B,r_k}\Gamma(m_{B,r_k})\Omega_{B,r_k}^{m_{B,r_k}}\eta^{m_{B,r_k}}}\right)\Bigg\}^K\Bigg]. \end{aligned} \quad (3.35)$$

Since $2m_{x,y}\Omega_{x,y}^{m_{x,y}}\eta^{m_{x,y}} - m_{x,y}^{m_{x,y}} z_{th}^{m_{x,y}} \gg m_{x,y}\Omega_{x,y}^{m_{x,y}}\eta^{m_{x,y}}$, $x, y \in \{A, B, r_k\}$, at high SNR region, the outage probability can be rewritten as

$$P_{out} \approx \frac{1}{2K}\sum_{k=1}^{K}\Bigg[\left\{\frac{2}{m_{A,r_k}\Gamma(m_{A,r_k})}\left(\frac{m_{A,r_k}z_{th}}{\Omega_{A,r_k}\eta}\right)^{m_{A,r_k}}\right\}^{K} + \left\{\frac{2}{m_{B,r_k}\Gamma(m_{B,r_k})}\left(\frac{m_{B,r_k}z_{th}}{\Omega_{B,r_k}\eta}\right)^{m_{B,r_k}}\right\}^{K}\Bigg]. \tag{3.36}$$

Let $m_{min} = \min\{m_{A,r_1}, \cdots, m_{A,r_K}, m_{B,r_1}, \cdots, m_{B,r_K}\}$. Then, the outage probability can be rewritten as

$$P_{out} \approx \frac{\eta^{-m_{min}K}}{2K}\sum_{k=1}^{K}\Bigg[\left\{\frac{2}{m_{A,r_k}\Gamma(m_{A,r_k})\eta^{m_{A,r_k}-m_{min}}}\left(\frac{m_{A,r_k}z_{th}}{\Omega_{A,r_k}}\right)^{m_{A,r_k}}\right\}^{K} + \left\{\frac{2}{m_{B,r_k}\Gamma(m_{B,r_k})\eta^{m_{B,r_k}-m_{min}}}\left(\frac{m_{B,r_k}z_{th}}{\Omega_{B,r_k}}\right)^{m_{B,r_k}}\right\}^{K}\Bigg]. \tag{3.37}$$

We can define the diversity order as [28]

$$d = \lim_{\eta\to\infty} -\frac{\log\left(P_{out}\right)}{\log\left(\eta\right)}. \tag{3.38}$$

From (3.37) and (3.38), the diversity order d is obtained as

$$d = K\min\{m_{A,r_1}, \cdots, m_{A,r_K}, m_{B,r_1}, \cdots, m_{B,r_K}\}. \tag{3.39}$$

Note that the diversity order depends on the number of relays and fading severity parameter $m_{i,j}$, $i, j \in \{A, B, r_1, \cdots, r_K\}$.

3.3 Performance Analysis of Reactive CDF-Based Relay Selection

3.3.1 Average Relay Fairness Anlaysis

Let random variables $U_{A,k} \triangleq F_{Z_{A,r_k}}(Z_{A,r_k})$ and $U_{B,k} \triangleq F_{Z_{B,r_k}}(Z_{B,r_k})$. Note that although the channels are i.ni.d., $U_{A,k}$ and $U_{B,k}$ are i.i.d. uniform random variables from 0 to 1 [73], [88]. Let $U_k = \min\{U_{A,k}, U_{B,k}\}$. Then, the probability of selecting r_k is given by

$$\begin{aligned}
\Pr(r^* = r_k) &= \Pr(r^* = r_k \,|\, r_k \in \mathcal{C}) \Pr(r_k \in \mathcal{C}) \\
&= \Pr(U_k = \max_{r_i \in \mathcal{C}} U_i, \, r_k \in \mathcal{C}) \\
&= \Pr(U_k = \max_{r_i \in \mathcal{C}} U_i, r_k \in \mathcal{C}, U_{A,k} < U_{B,k}) + \Pr(U_k = \max_{r_i \in \mathcal{C}} U_i, r_k \in \mathcal{C}, U_{A,k} \geq U_{B,k}) \\
&= \iint\limits_{\mathcal{A}_9 \cap \mathcal{A}_{11}} \Pr(x = \max_{r_i \in \mathcal{C}} U_i \,|\, U_{A,k} = x, U_{B,k} = y) f_{U_{A,k}}(x) f_{U_{B,k}}(y) dx dy \\
&\quad + \iint\limits_{\mathcal{A}_{10} \cap \mathcal{A}_{11}} \Pr(y = \max_{r_i \in \mathcal{C}} U_i \,|\, U_{A,k} = x, U_{B,k} = y) f_{U_{B,k}}(y) f_{U_{A,k}}(x) dy dx \qquad (3.40)
\end{aligned}$$

where $\mathcal{A}_9 = \{(x,y) \,|\, x < y\}$, $\mathcal{A}_{10} = \{(x,y) \,|\, x \geq y\}$, and $\mathcal{A}_{11} = \{(x,y) \,|\, x \geq F_{U_{A,k}}(z_{th}), \, y \geq F_{U_{B,k}}(z_{th})\}$. Note that the relay r_k belongs to the decoding set $\mathcal{C}$ if and only if both $U_{A,k}$ and $U_{B,k}$ belong to region $\mathcal{A}_{11}$. The probability of selecting

r_k is rewritten as

$$\Pr(r^*=r_k)=\int_{\omega_k}^{1}\int_{\nu_{A,k}}^{y}\underbrace{\Pr(x=\max_{r_i\in\mathcal{C}}U_i|U_{A,k}=x,U_{B,k}=y)}_{\Xi_1}f_{U_{A,k}}(x)f_{U_{B,k}}(y)dxdy$$
$$+\int_{\omega_k}^{1}\int_{\nu_{B,k}}^{x}\underbrace{(y=\max_{r_i\in\mathcal{C}}U_i|U_{A,k}=x,U_{B,k}=y)}_{\Xi_2}f_{U_{B,k}}(y)f_{U_{A,k}}(x)dydx \quad (3.41)$$

where $\omega_k=\max\{F_{Z_{A,r_k}}(z_{th}),\ F_{Z_{B,r_k}}(z_{th})\}$, $\nu_{A,k}=F_{Z_{A,r_k}}(z_{th})$, and $\nu_{B,k}=F_{Z_{B,r_k}}(z_{th})$. Since a relay is selected such that $r^*=\arg\max_{r_k\in\mathcal{C}}\{\min\{U_{A,k},U_{B,k}\}\}$, Ξ_1 in (3.41) is given by

$$\Xi_1=\prod_{\substack{r_i\in\mathcal{C}\\ r_i\neq r_k}}\Pr(\min\{U_{A,i},U_{B,i}\}<x\,|\,U_{A,k}=x,U_{B,k}=y). \quad (3.42)$$

The number of subsets which have cardinality $l-1$ among all subsets of $\mathcal{R}\backslash\{r_k\}$ is $\binom{K-1}{l-1}$. Let $\mathcal{R}_{l,n}^{k}$ denote the nth subset among them, $n=1,\ 2,\ \cdots,\ \binom{K-1}{l-1}$. Then, Ξ_1 is rewritten as

$$\Xi_1=\sum_{l=1}^{K}\sum_{n=1}^{\binom{K-1}{l-1}}\Pr(\mathcal{C}\backslash\{r_k\}=\mathcal{R}_{l,n}^{k})\prod_{r_i\in\mathcal{R}_{l,n}^{k}}\Pr(U_i<x\,|\,\mathcal{C}\backslash\{r_k\}=\mathcal{R}_{l,n}^{k},U_{A,k}=x,U_{B,k}=y). \quad (3.43)$$

The probability that the set $\mathcal{C}\backslash\{r_k\}$ becomes the subset $\mathcal{R}_{l,n}^{k}$ is given by

$$\Pr(\mathcal{C}\backslash\{r_k\}=\mathcal{R}_{l,n}^{k})=\prod_{r_i\in\mathcal{R}_{l,n}^{k}}\Pr(r_i\in\mathcal{C})\prod_{r_j\notin\mathcal{R}_{l,n}^{k}\cup\{r_k\}}\Pr(r_j\notin\mathcal{C}). \quad (3.44)$$

From (3.43) and (3.44), Ξ_1 is given by

$$\Xi_1=\sum_{l=1}^{K}\sum_{n=1}^{\binom{K-1}{l-1}}\prod_{r_i\in\mathcal{R}_{l,n}^{k}}\Pr(r_i\in\mathcal{C})\Pr(U_i<x|r_i\in\mathcal{C},U_{A,k}=x,U_{B,k}=y)$$
$$\times\prod_{r_j\notin\mathcal{R}_{l,n}^{k}\cup\{r_k\}}\Pr(r_j\notin\mathcal{C}). \quad (3.45)$$

Assume that if the received SNR at the relay is larger than the threshold z_{th}, the relay successfully decodes its received signal. Then, the probability that r_i belongs to $\mathcal{C}$ is given by

$$\begin{aligned}\Pr(r_i \in \mathcal{C}) &= \Pr(Z_{A,r_i} \geq z_{th}) \Pr(Z_{B,r_i} \geq z_{th}) \\ &= (1-\nu_{A,i})(1-\nu_{B,i}). \end{aligned} \tag{3.46}$$

The probability that U_i is smaller than x conditioned on $r_i \in \mathcal{C}$, $U_{A,k} = x$, and $U_{B,k} = y$ is given by

$$\begin{aligned}&\Pr(U_i < x \,|\, r_i \in \mathcal{C},\, U_{A,k} = x,\, U_{B,k} = y) \\ &\qquad = 1 - \Pr(\min\{U_{A,i}, U_{B,i}\} > x \,|\, r_i \in \mathcal{C}) \\ &\qquad = 1 - \frac{\Pr(U_{A,i} \geq x, U_{B,i} \geq x,\, r_i \in \mathcal{C} \,|\, U_{A,k} = x,\, U_{B,k} = y)}{\Pr(r_i \in \mathcal{C})} \\ &\qquad = 1 - \frac{(1-\max\{x, \nu_{A,i}\})(1-\max\{x,\nu_{B,i}\})}{(1-\nu_{A,i})(1-\nu_{B,i})}. \end{aligned} \tag{3.47}$$

From (3.45), (3.46), and (3.47), Ξ_1 is rewritten as

$$\begin{aligned}\Xi_1 = \sum_{l=1}^{K} \sum_{n=1}^{\binom{K-1}{l-1}} \prod_{r_i \in \mathcal{R}_{l,n}^k} &\{(1-\nu_{A,i})(1-\nu_{B,i}) - (1-\max\{x,\nu_{A,i}\})(1-\max\{x,\nu_{B,i}\})\} \\ &\times \prod_{r_j \notin R_{l,n}^k \cup \{r_k\}} \{1-(1-\nu_{A,j})(1-\nu_{B,j})\}. \end{aligned} \tag{3.48}$$

For Ξ_2, similar result is obtained using same procedure. Ξ_2 in (3.41) is given by

$$\begin{aligned}\Xi_2 = \sum_{l=1}^{K} \sum_{n=1}^{\binom{K-1}{l-1}} \prod_{r_i \in \mathcal{R}_{l,n}^k} &\{(1-\nu_{A,i})(1-\nu_{B,i}) - (1-\max\{y,\nu_{A,i}\})(1-\max\{y,\nu_{B,i}\})\} \\ &\times \prod_{r_j \notin R_{l,n}^k \cup \{r_k\}} \{1-(1-\nu_{A,j})(1-\nu_{B,j})\}. \end{aligned} \tag{3.49}$$

Plugging (3.48) and (3.49) into (3.41), the probability of selecting r_k is given by

$$\Pr(r^* = r_k) = \sum_{l=1}^{K} \sum_{n=1}^{\binom{K-1}{l-1}} \prod_{r_j \notin \mathcal{R}_{l,n}^k \cup \{r_k\}} \{1 - (1-\nu_{A,j})(1-\nu_{B,j})\}$$
$$\times \left[\int_{\omega_k}^{1} \int_{\nu_{A,k}}^{y} \prod_{r_i \in \mathcal{R}_{l,n}^k} \{(1-\nu_{A,i})(1-\nu_{B,i}) - (1-\max\{x,\nu_{A,i}\})(1-\max\{x,\nu_{B,i}\})\} \right.$$
$$\times f_{U_{A,k}}(x)dx f_{U_{B,k}}(y)dy$$
$$+ \int_{\omega_k}^{1} \int_{\nu_{B,k}}^{x} \prod_{r_i \in \mathcal{R}_{l,n}^k} \{(1-\nu_{A,i})(1-\nu_{B,i}) - (1-\max\{y,\nu_{A,i}\})(1-\max\{y,\nu_{B,i}\})\}$$
$$\left. \times f_{U_{B,k}}(y)dy f_{U_{A,k}}(x)dx \right]. \tag{3.50}$$

Deriving the closed-form expression of (3.50) is difficult due to the max operation. Instead, to get insight for the average relay fairness, we derive asymptotic relay selection probability. When $\bar{z}_{i,j}$ is high, the incomplete gamma function can be approximated as [61, eq. (8.354.1)]

$$\gamma\left(m_{i,j}, \frac{m_{i,j}x}{\bar{z}_{i,j}}\right) \approx \frac{1}{m_{i,j}} \left(\frac{m_{i,j}x}{\bar{z}_{i,j}}\right)^{m_{i,j}}. \tag{3.51}$$

From (3.2) and (3.51), it is clear that as $\bar{z}_{i,j}$ increases, $\nu_{A,k}$, $\nu_{B,k}$, and ω_k go to zero. Moreover, $\Pr(r_i \in \mathcal{C})$ goes to 1 and the decoding set becomes potential relay set $\mathcal{R}$. Then, the probability of selecting r_k in (3.50) can be approximated as

$$\Pr(r^* = r_k) \approx \int_0^1 \int_0^y (2x - x^2)^{K-1} f_{U_{A,k}}(x)dx f_{U_{B,k}}(y)dy$$
$$+ \int_0^1 \int_0^x (2y - y^2)^{K-1} f_{U_{B,k}}(y)dy f_{U_{A,k}}(x)dx. \tag{3.52}$$

By using [61, eq. (3.194.1)], [61, eq. (7.512.11)], and [62, eq. (15.1.26)], (3.52) can be rewritten as

$$
\begin{aligned}
\Pr(r^* = r_k) &\approx \frac{2^K}{K} \int_0^1 y^K {}_2F_1 \left(1-K, K; 1+K; \frac{y}{2}\right) dy \\
&\approx \frac{2^K}{K} \frac{\Gamma(K+1)}{\Gamma(K+2)} {}_2F_1 \left(1-K, K; K+2; \frac{1}{2}\right) \\
&\approx \frac{1}{K}.
\end{aligned} \tag{3.53}
$$

Note that at high SNR region, relay selection probability does not depend on the types of fading channels, but only on the number of relays. From (2.11), (2.10), and (3.53), it is clear that as ζ_{r_k} goes to $1/K$, average relay fairness $\mathcal{F}$ becomes 1, which means that strict fairness for relays is achieved.

3.3.2 Outage Probability Analysis

An outage occurs when the decoding set is empty or the SNR at either user A or user B from the selected relay is smaller than the SNR threshold z_{th}. Define a random variable $Z_{r_k} \triangleq \min\{Z_{r_k,A},\, Z_{r_k,B}\}$. Then, the outage probability is given by

$$
P_{out} = \sum_{k=1}^{K} \Pr(Z_{r_k} < z_{th}, r^* = r_k) + \Pr(\mathcal{C} = \emptyset). \tag{3.54}
$$

Under the assumption that $P_A = P_B = P_{r_1} = \cdots = P_{r_K}$, since $Z_{i,r_k} = Z_{r_k,i}$ and $Z_{i,r_k} \geq z_{th}$, $i \in \{A, B\}$, the first term on the right-hand side of (3.54) is zero. Hence, the outage probability can be rewritten as

$$\begin{aligned}
P_{out} &= \Pr(\mathcal{C} = \emptyset) \\
&= \prod_{k=1}^{K} (\Pr(Z_{A,r_k} < z_{th}) + \Pr(Z_{B,r_k} < z_{th}) - \Pr(Z_{A,r_k} < z_{th}) \Pr(Z_{B,r_k} < z_{th})) \\
&= \prod_{k=1}^{K} (1 - \Pr(Z_{A,r_k} \geq z_{th}) \Pr(Z_{B,r_k} \geq z_{th})) \\
&= \prod_{k=1}^{K} \left(1 - \frac{\Gamma\left(m_{A,r_k}, \frac{m_{A,r_k} z_{th}}{\bar{\gamma}_{A,r_k}}\right) \Gamma\left(m_{B,r_k}, \frac{m_{B,r_k} z_{th}}{\bar{\gamma}_{B,r_k}}\right)}{\Gamma(m_{A,r_k}) \Gamma(m_{B,r_k})} \right). \qquad (3.55)
\end{aligned}$$

Note that in this network, the reactive schemes where a relay is selected in the decoding set have the same outage probability although they may select different relays having different received SNRs. When outage probability is analyzed, practical coding scheme is not considered and it is assumed that if the received SNR at a node is larger than SNR threshold, the node successfully decodes its received signal [28], [64]. Then, coding gain is obtained from an outage probability [108], [109]. When, at high SNR region, an outage probability is written as $P_{out} \approx (O_C \gamma)^{-O_D}$ where γ is the average SNR, then O_D is the diversity gain and O_C is the coding gain [108], [109]. Therefore, if the outage probabilities of two arbitrary schemes are same, then coding gains of two schemes are also same. Since the reactive schemes where a relay is selected in the decoding set have the same outage probability, they also have same coding gain.

From (3.51) and (3.55), the outage probability can be approximated as

$$P_{out} \approx \prod_{k=1}^{K} \left\{ 1 - \left(1 - \frac{\left(\frac{m_{r_k} z_{th}}{\bar{z}_{A,r_k}}\right)^{m_{A,r_k}}}{\Gamma(m_{A,r_k}) m_{A,r_k}} \right) \left(1 - \frac{\left(\frac{m_{B,r_k} z_{th}}{\bar{z}_{B,r_k}}\right)^{m_{B,r_k}}}{\Gamma(m_{B,r_k}) m_{B,r_k}} \right) \right\}. \qquad (3.56)$$

Let $m_k = \min\{m_{A,r_k}, m_{B,r_k}\}$ and $\eta = P/N_0$. Then, the outage probability can be rewritten as

$$P_{out} \approx \prod_{k=1}^{K} \frac{1}{\eta^{m_k}} \left\{ \left(\frac{(m_{A,r_k} z_{th})^{m_{A,r_k}}}{\Omega_{A,r_k}^{m_{A,r_k}} \eta^{m_{A,r_k}-m_k} \Gamma(m_{A,r_k}) m_{A,r_k}} \right) + \left(\frac{(m_{B,r_k} z_{th})^{m_{B,r_k}}}{\Omega_{B,r_k}^{m_{B,r_k}} \eta^{m_{B,r_k}-m_k} \Gamma(m_{B,r_k}) m_{B,r_k}} \right) - \left(\frac{(m_{A,r_k} z_{th})^{m_{A,r_k}} (m_{B,r_k} z_{th})^{m_{B,r_k}}}{\bar{z}_{A,r_k}^{m_{A,r_k}} \bar{z}_{B,r_k}^{m_{B,r_k}} \eta^{-m_k} \Gamma(m_{A,r_k}) \Gamma(m_{B,r_k}) m_{A,r_k} m_{B,r_k}} \right) \right\}. \tag{3.57}$$

We can define the diversity order as [28]

$$d = \lim_{\eta \to \infty} -\frac{\log(P_{out})}{\log(\eta)}. \tag{3.58}$$

From (3.57) and (3.58), the diversity order is obtained as

$$d = \sum_{k=1}^{K} \min\{m_{A,r_k}, m_{B,r_k}\}. \tag{3.59}$$

Note that the diversity order depends on the number of relays and fading severity parameter $m_{i,j}$.

3.4 Numerical Results

Consider a two-way relay network consisting of two users, A and B, and three relays. We assume that $\Omega_{i,j} = \mathrm{d}_{i,j}^{-3}$ where $\mathrm{d}_{i,j}$ is the distance between node i and node j. Assume that the noise variance N_0 is 1. We will use the notations $\mathbf{m}_{A,r_k} = \{m_{A,r_1}, m_{A,r_2}, \cdots, m_{A,r_K}\}$ and $\mathbf{m}_{B,r_k} = \{m_{B,r_1}, m_{B,r_2}, \cdots, m_{B,r_K}\}$. To compare the performance of the proposed schemes, max-min SNR-based relay selection in [98]-[107] and proportional fair relay selection (which is relay selection based on the relative instantaneous-to-average value of SNR) are presented.

3.4.1 Average Relay Fairness

Fig. 3.3 shows the average relay fairness of various proactive relay selection schemes with various fading severity parameters for $K = 3$. It is shown that the proactive CDF-based relay selection scheme achieves 1 regardless of the SNR. It is shown that the proactive CDF-based relay selection scheme achieves higher average fairness than any other relay selection schemes.

Fig. 3.4 show the average relay fairness of reactive CDF-based relay selection scheme with various fading severity parameters for $K = 3$. For simplicity, we will use the notation $m = m_{A,r_k} = m_{B,r_k}$ in Fig. 3.4(a). It is shown that the analytical results of the reactive CDF-based relay selection scheme perfectly match the simulation results. Fig. 3.4(a) shows that as the value of parameter increases, the average relay fairness of the reactive CDF-based relay selection scheme goes to 1 quickly. Fig. 3.4(b) shows that for $\eta \geq 12$ dB, the higher average relay fairness is achieved when $\mathbf{m}_{A,r_k} = \{1, 2, 3\}$ and $\mathbf{m}_{B,r_k} = \{1, 2, 3\}$.

Fig. 3.5 shows the average relay fairness of various reactive relay selection schemes with various fading severity parameters for $K = 3$. It is shown that as the SNR increases, the average relay fairness of the the reactive CDF-based relay selection scheme increases and goes to 1 nevertheless the channels experience different fading. On the other hand, the reactive proportional fair relay selection scheme and the reactive max-min SNR-based relay selection scheme do not achieve relay fairness strictly. It is shown that the reactive CDF-based relay selection scheme achieves higher average relay fairness than the reactive random relay selection scheme. In reactive random relay selection scheme, since the relays in the decoding set have same selection probability, a relay having lower successful decoding probability is selected less frequently than other relays. Whereas, in reactive CDF-based relay selection, if a relay having lower successful decoding probability belongs to the decoding set, the relay is more likely to be selected because it may have higher value of the CDF. Detailed explanation about the comparison with reactive random relay selection scheme and reactive CDF-based relay selection Scheme is as follows.

Comparison with Reactive Random Relay Selection Scheme and Reactive CDF-Based Relay Selection Scheme

Guaranteeing the relay fairness strictly means that all relays have equal selection probability, that is, $\Pr(r^* = r_k) = 1/K, \forall k$, where K is total number of relays. When the difference between the selection probabilities of relays is small, it means that high fairness is achieved. When the difference between the selection probabilities of relays is small, it means that high fairness is achieved. For example, we assume that there are three relays, r_1, r_2, r_3. If we assume that by using 'relay selection scheme A', the selection probabilities of relays r_1, r_2, and r_3 are 0.1, 0.3, 0.6 (i.e. $\Pr(r^* = r_1) = 0.1$, $\Pr(r^* = r_2) = 0.3$, $\Pr(r^* = r_3) = 0.6$), respectively, and by using 'relay selection scheme B', the selection probabilities of relays r_1, r_2, and r_3 are 0.25, 0.35, 0.4 (i.e. $\Pr(r^* = r_1) = 0.25$, $\Pr(r^* = r_2) = 0.35$, $\Pr(r^* = r_3) = 0.4$), respectively, then 'relay selection scheme B' achieves higher fairness than 'relay selection scheme A'.

In reactive random relay selection scheme, the relays in the decoding set $\mathcal{C}$ are selected with equal probability $1/|\mathcal{C}|$ among relays in the decoding set where $|\mathcal{C}|$ is cardinality of $\mathcal{C}$. Therefore, the selection probability can be simply written as

$$\begin{aligned}\Pr(r^* = r_k) &= \Pr(r_k \in \mathcal{C}) \Pr(r^* = r_k \,|\, r_k \in \mathcal{C}) \\ &= \Pr(r_k \in \mathcal{C}) \frac{1}{|\mathcal{C}|}.\end{aligned} \tag{3.60}$$

In reactive CDF-based relay selection scheme, since a relay in the decoding set is selected by comparing the values of CDFs of SNRs at relays, the relays in the decoding set are selected in the different probabilities among the relays in the decoding set. The selection probability can be simply written as

$$\begin{aligned}\Pr(r^* = r_k) &= \Pr(r_k \in \mathcal{C}) \Pr(r^* = r_k \,|\, r_k \in \mathcal{C}) \\ &= \Pr(r_k \in \mathcal{C}) Pr(r_k = \arg\max_{r_i \in \mathcal{C}} F_{Z_{r_i}}(Z_{r_i})).\end{aligned} \tag{3.61}$$

As you see eq. (3.60) and eq. (3.61), the selection probabilities of relay r_k by using two schemes are different each other.

When a certain decoding set is given, in reactive random relay selection scheme, since $\Pr(r^* = r_k \,|\, r_k \in \mathcal{C})$ has constant value $1/|\mathcal{C}|$, the selection probability of r_k, $\Pr(r^* = r_k)$, is proportional to the successful decoding probability, $\Pr(r_k \in \mathcal{C})$. That is, the relay having lower successful decoding probability, $\Pr(r_k \in \mathcal{C})$, has lower selection probability, $\Pr(r^* = r_k)$. Otherwise, in reactive CDF-based relay selection scheme, the selection probability in the decoding set, $\Pr(r^* = r_k \,|\, r_k \in \mathcal{C})$, is affected by the successful decoding probability, $\Pr(r_k \in \mathcal{C})$. The relay having lower successful decoding probability has the higher selection probability in the decoding set. So, the relay having lower successful decoding probability may not have lower selection probability. It will be explained through the example below. As you see the Fig.

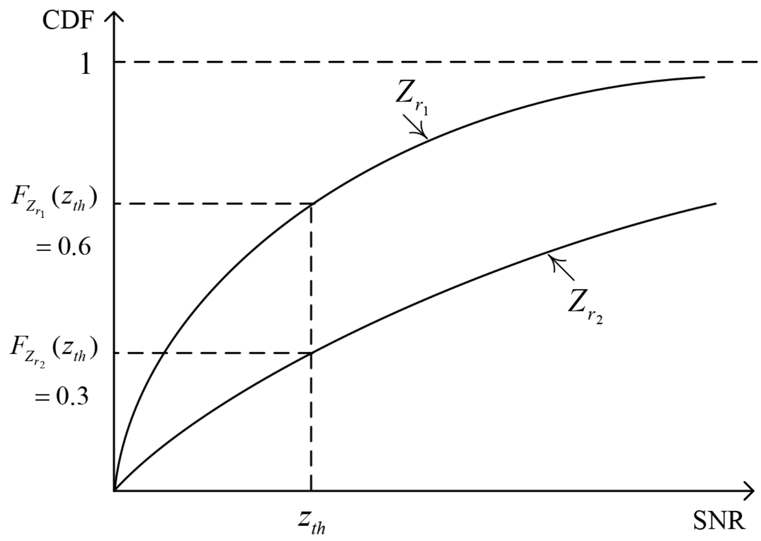
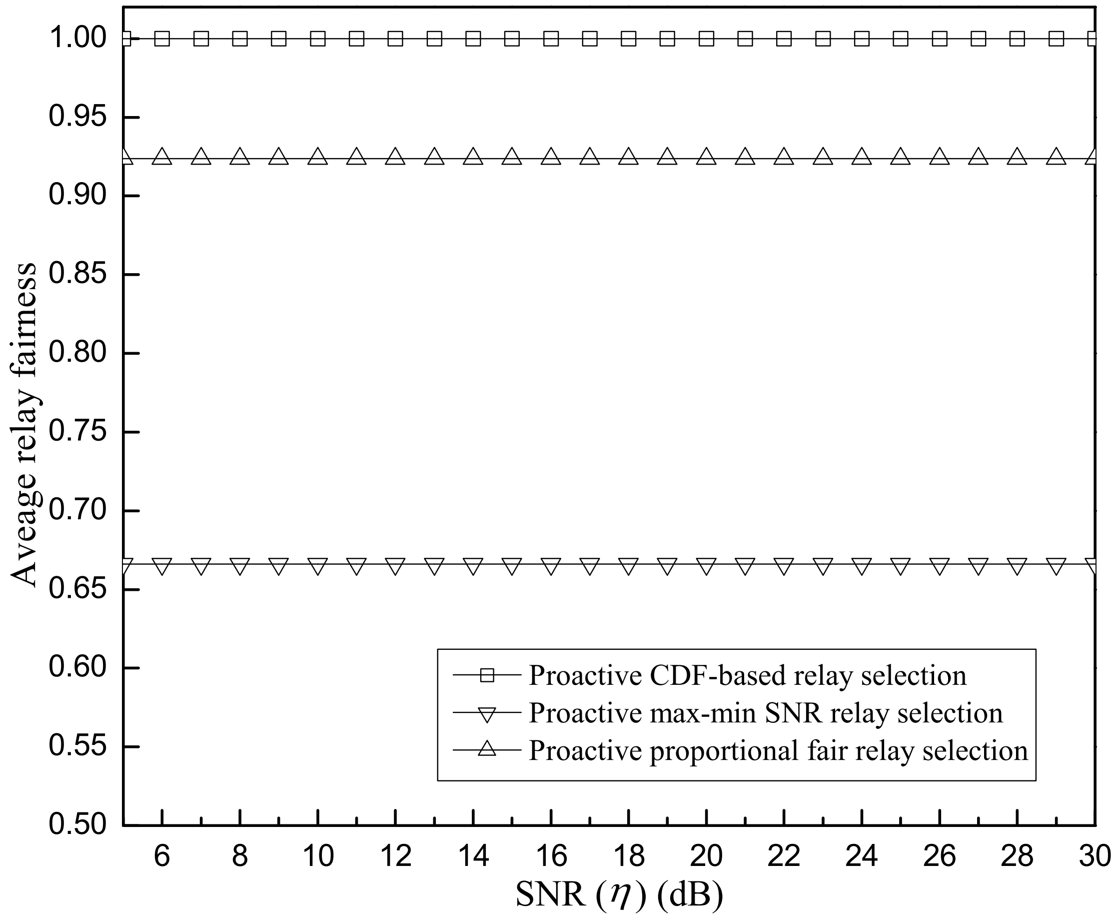


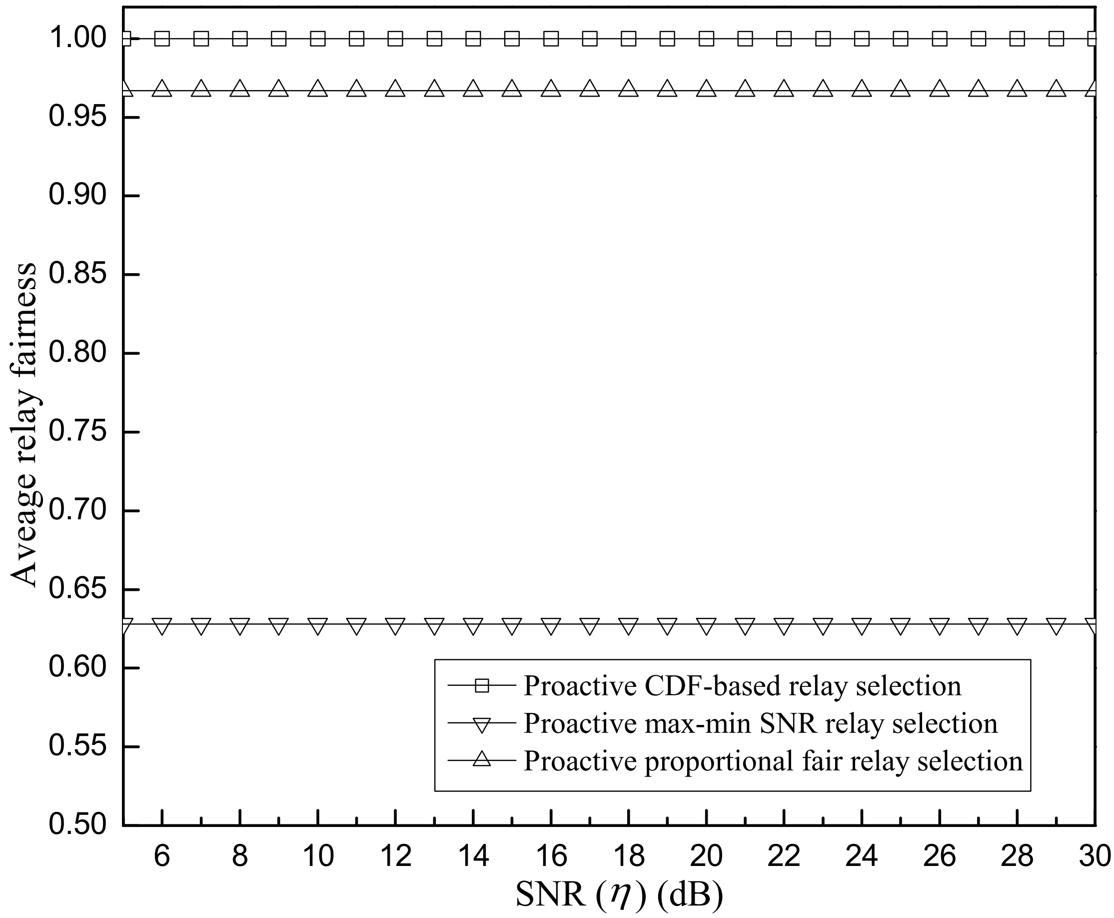
Figure 3.2. CDF of SNR.

3.2, the successful decoding probability of relay r_1 is given by $\Pr(Z_{r_1} \geq z_{th}) = 1 - F_{Z_{r_1}}(z_{th}) = 0.4$, and similarly, the successful decoding probability of the relay r_2 is 0.7. So, the relay having higher successful decoding probability is relay r_2. When relay r_1 and relay r_2 belong to the decoding set, the value of CDF of SNR at relay r_1 is uniformly distributed between 0.6 and 1.0, and the value of CDF of SNR at relay r_2 is uniformly distributed between 0.3 and 1.0. Intuitively, when relay r_1 and relay r_2 are in the decoding set, the probability that the value of CDF of SNR at relay r_1 is larger than that at relay r_2 is higher than the probability of the opposite case.

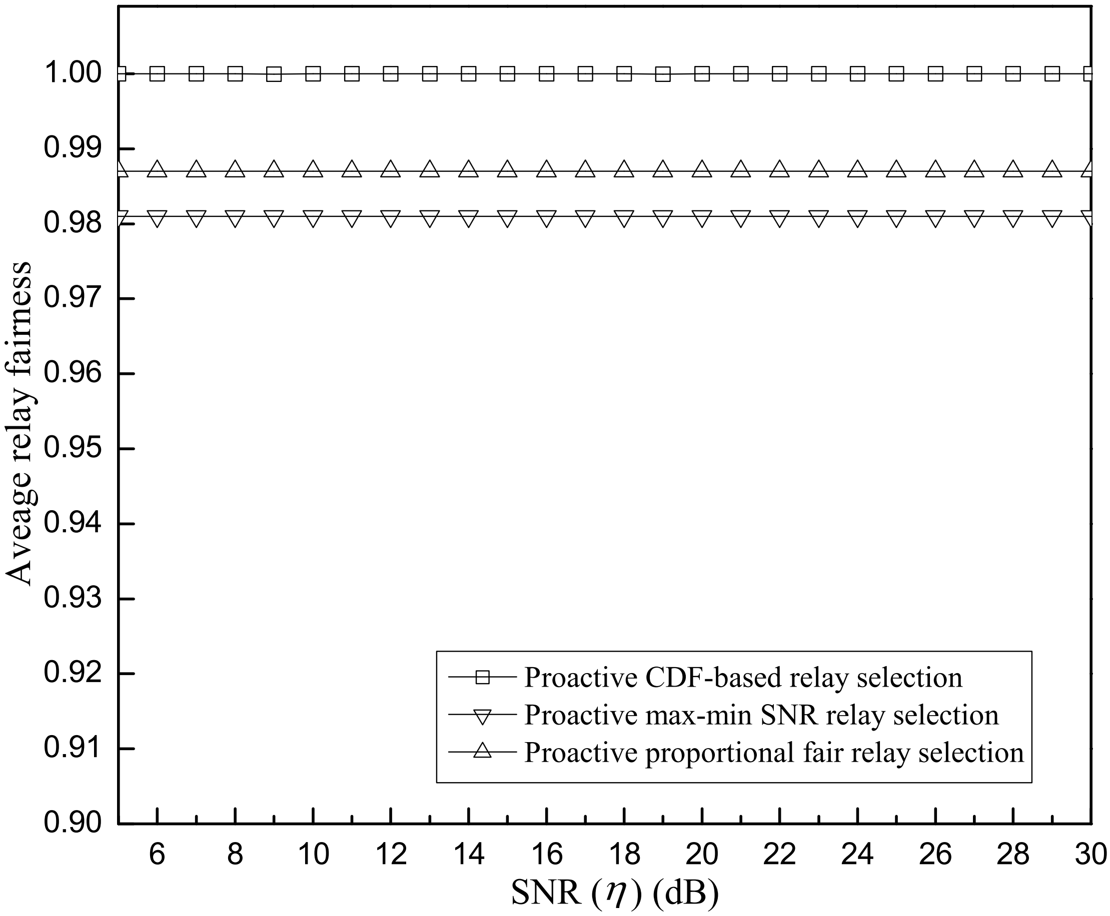
By this reason, we can expect that the difference between the selection probabilities of relays by using the reactive CDF-based relay selection scheme is smaller than that by using the reactive random relay selection scheme.



(a) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{0.5, 1.0, 2.0\}$

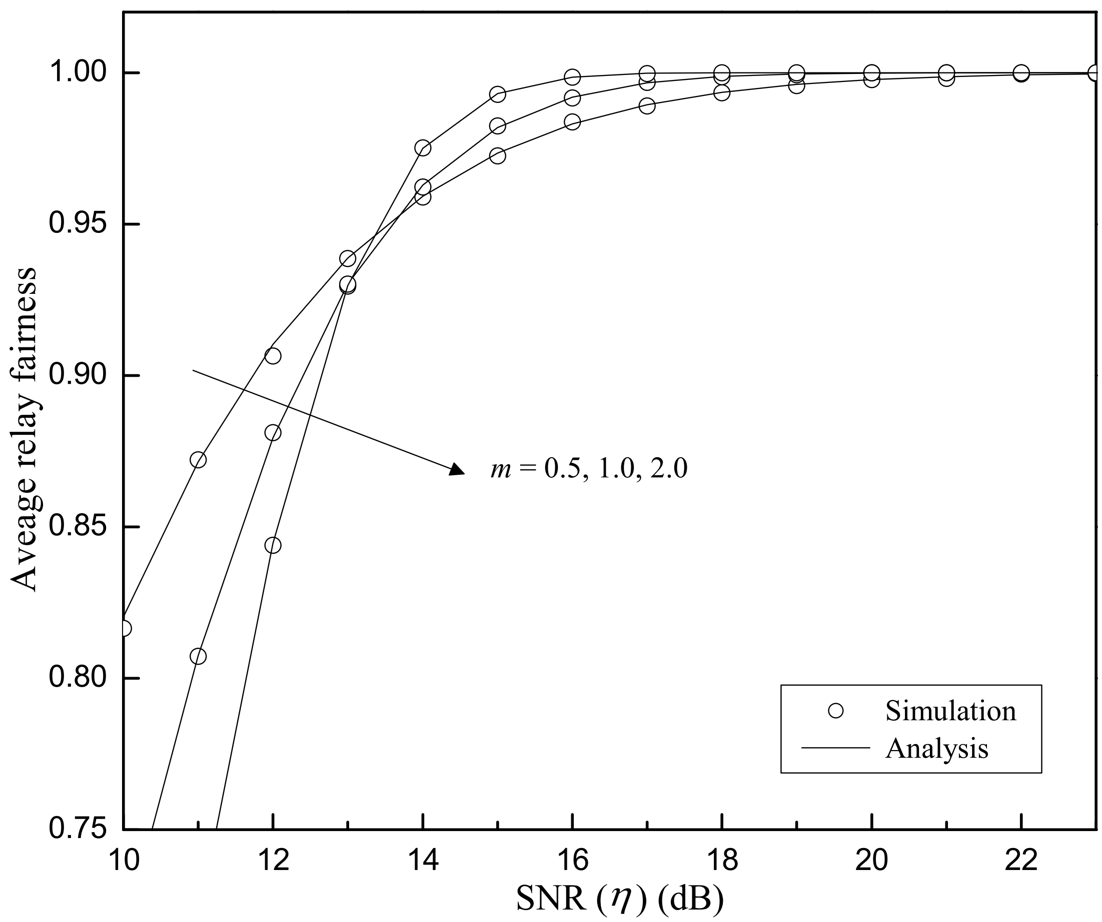


(b) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{1.0, 2.0, 3.0\}$



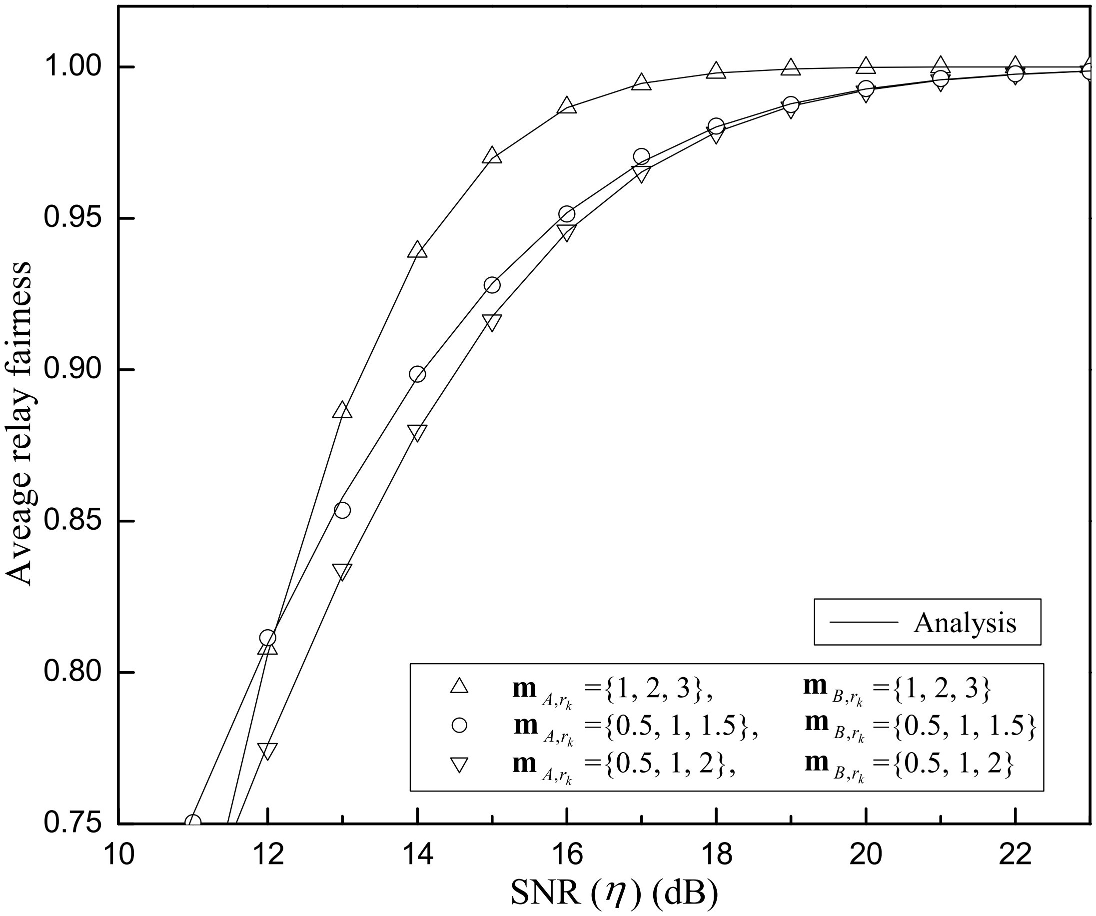
(c) $\mathbf{m}_{A,r_k} = \{0.5, 1.0, 1.5\}$ and $\mathbf{m}_{B,r_k} = \{3.0, 2.0, 0.5\}$

Figure 3.3. Average relay fairness of various proactive relay selection schemes.



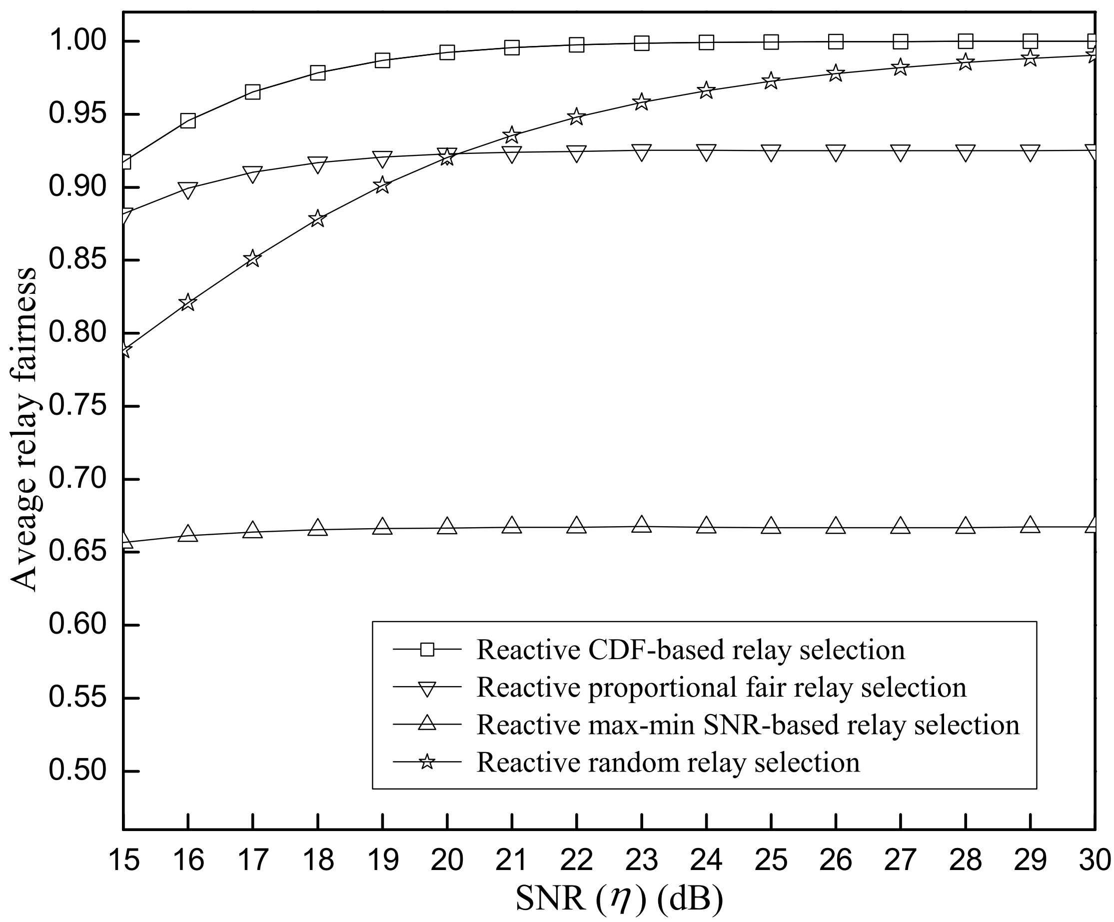
1.00
0.95
0.90
0.85
0.80
0.75
10
12
14
16
18
20
22
Aveage relay fairness
SNR (η) (dB)
m = 0.5, 1.0, 2.0
Simulation
Analysis

(a) $m_{A,r_k} = m_{B,r_k} = m$

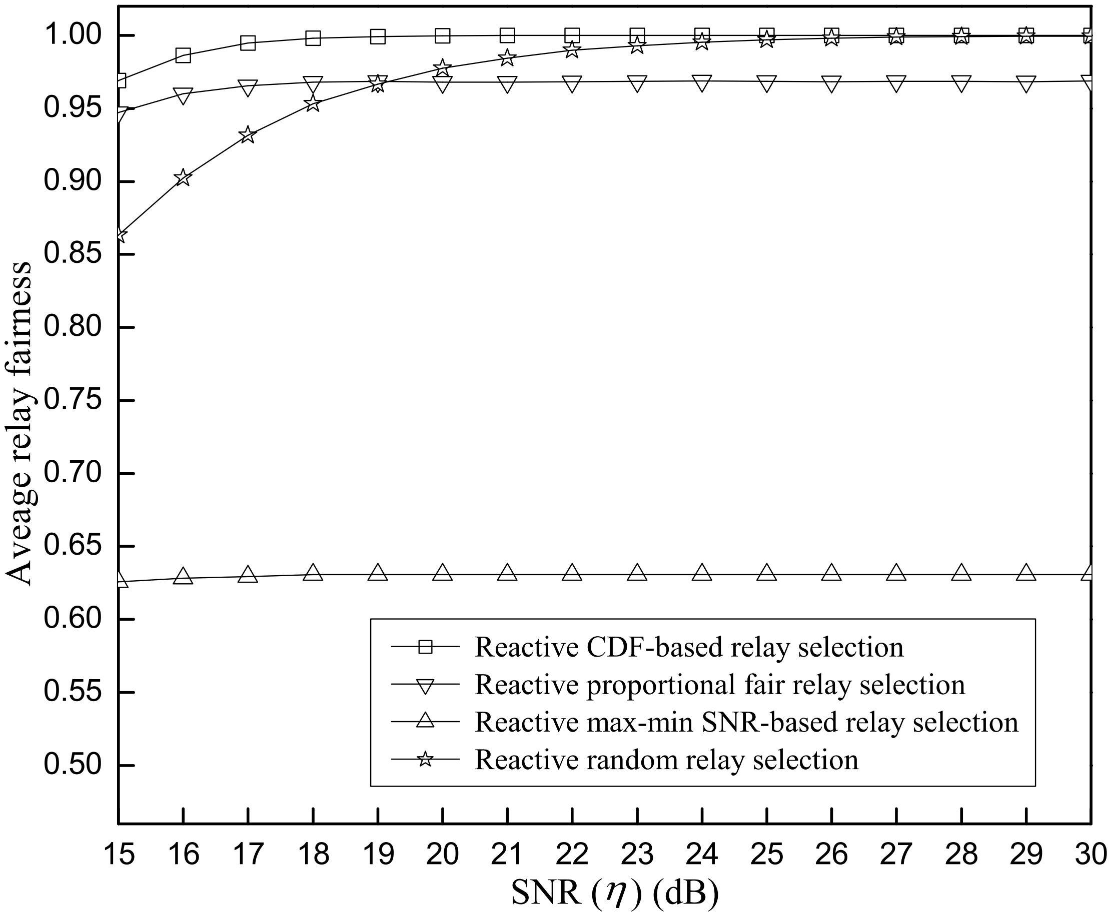


(b) Various values of m_{A,r_k} and m_{B,r_k}

Figure 3.4. Average relay fairness of reactive CDF-based relay selection scheme.



(a) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{0.5, 1.0, 2.0\}$



(b) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{1.0, 2.0, 3.0\}$

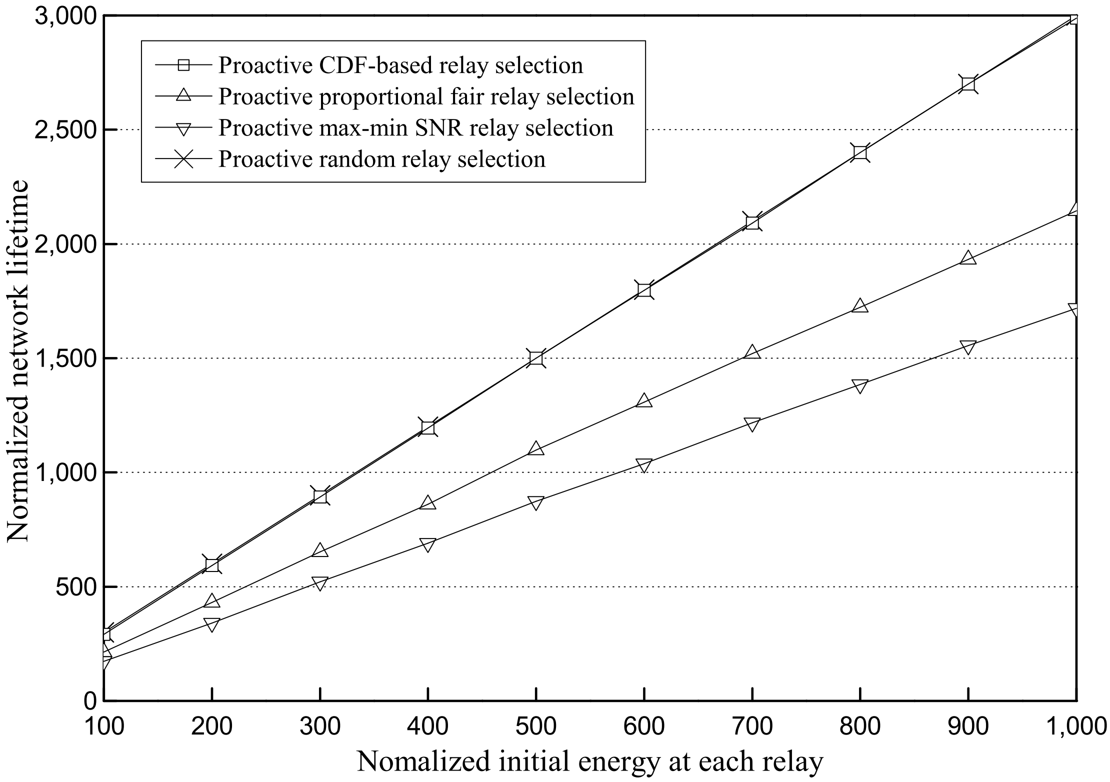
Figure 3.5. Average relay fairness of various reactive relay selection schemes.

3.4.2 Network Lifetime

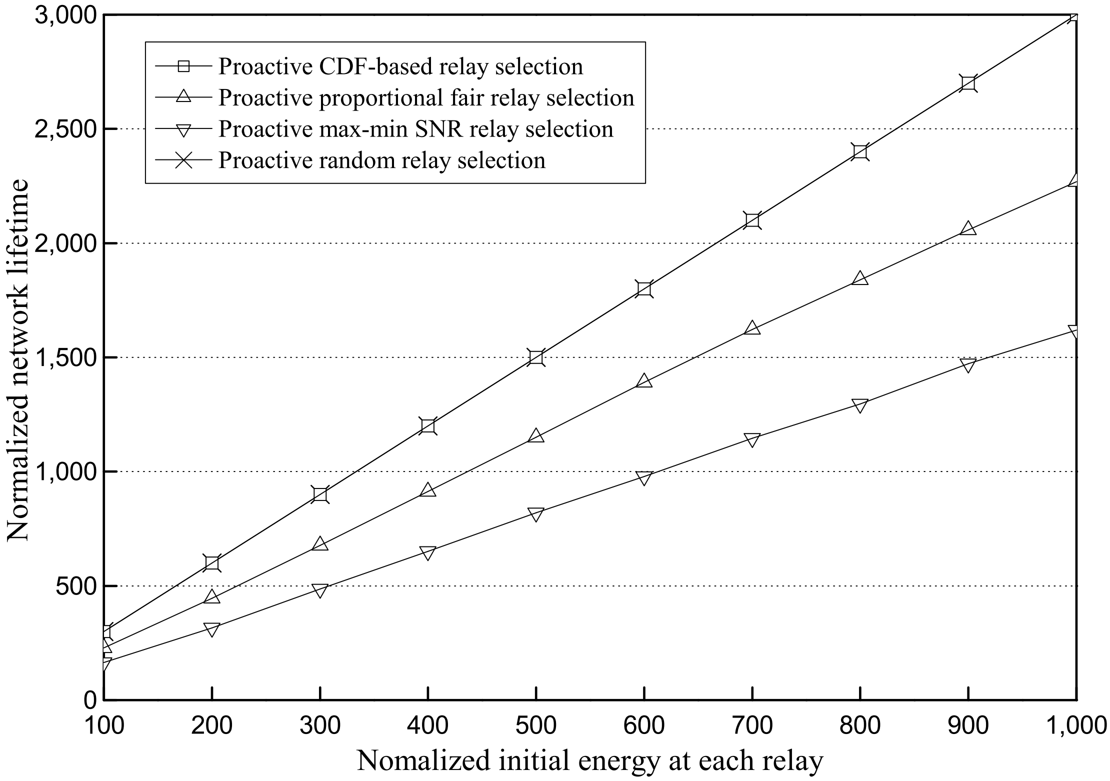
To provide insights into the impact of the average relay fairness on network lifetime, we will show network lifetime performances. Network lifetime is commonly defined as the time duration in which all nodes in the network remain active [90]-[94]. Suppose that the number of relays is three and the SNR is 20 dB. Suppose that $\{\Omega_{A,r_i}\}_{i=1}^{3} = \{\Omega_{B,r_i}\}_{i=1}^{3} = \{(1.2)^{-3}, (1.1)^{-3}, (1.0)^{-3}\}$.

Fig. 3.6 shows the network lifetime of various proactive relay selection schemes with various fading severity parameters. In figures, normalized initial energy at each relay means the initial energy when we assume that energy consumption for transmitting one packet is 1. Normalized network lifetime means the number of transmitted packet until one relay becomes inactive when we assume that the duration for transmitting one packet is 1. It is shown that proactive CDF-based relay selection scheme achieves higher network lifetime than other schemes except proactive random relay selection scheme. The reason that proactive CDF-based relay selection scheme and proactive random relay selection scheme achieve network lifetime is that they achieve same average relay fairness.

Fig. 3.7 shows the network lifetime of various reactive relay selection schemes with various fading severity parameters. It is shown that reactive CDF-based relay selection scheme achieves higher network lifetime than other schemes. Difference between the reactive CDF-based relay selection scheme and reactive proportional fair relay selection scheme for $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{0.5, 1.0, 2.0\}$ is larger than that for $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{1.0, 2.0, 3.0\}$.

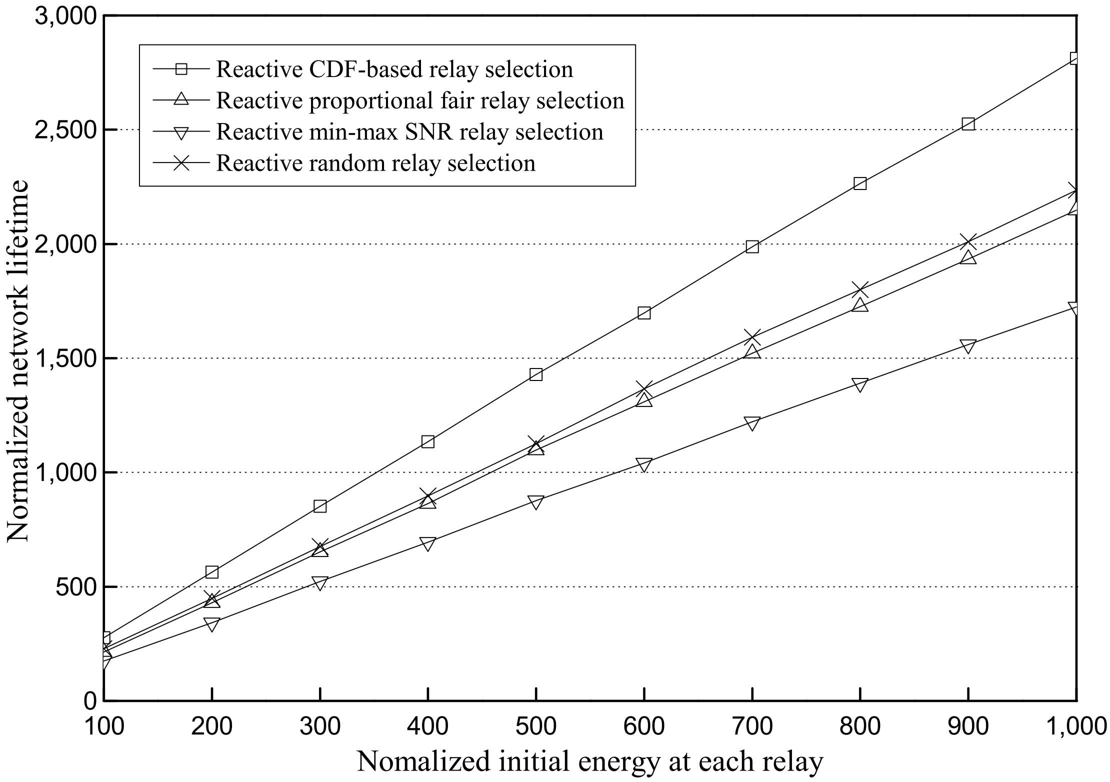


(a) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{0.5, 1.0, 1.5\}$

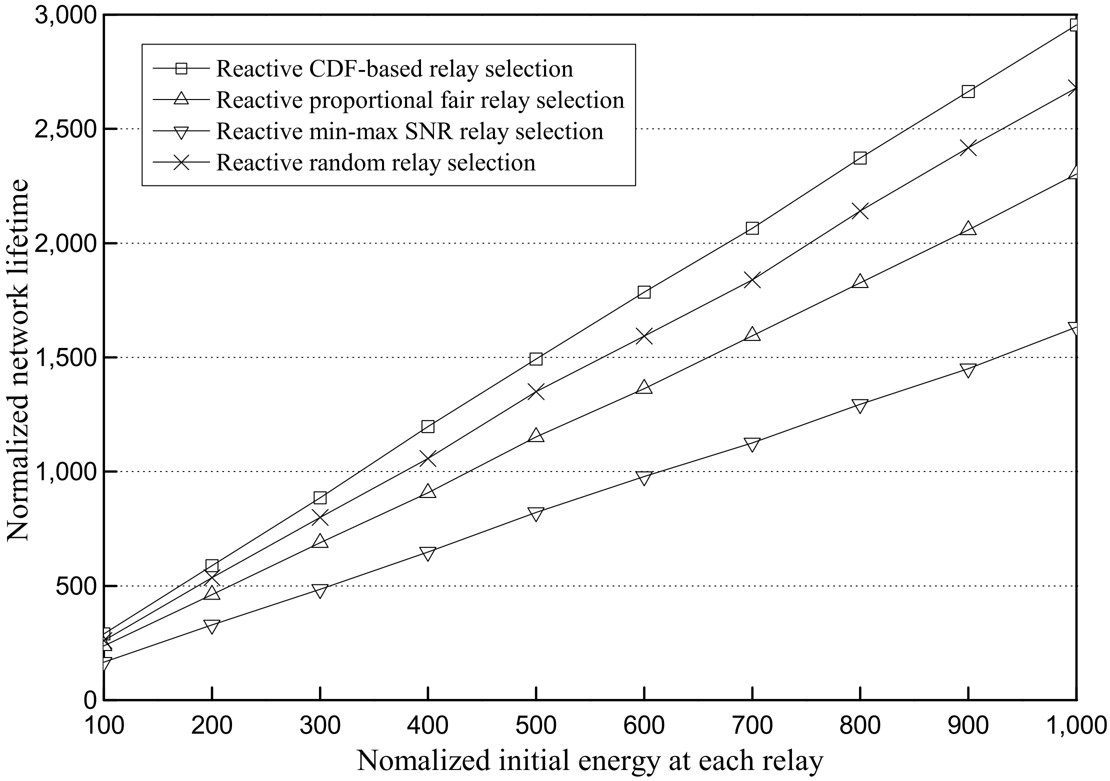


(b) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{1.0, 2.0, 3.0\}$

Figure 3.6. Network lifetime of various proactive relay selection schemes.



(a) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{0.5, 1.0, 1.5\}$



(b) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{1.0, 2.0, 3.0\}$

Figure 3.7. Network lifetime of various reactive relay selection schemes.

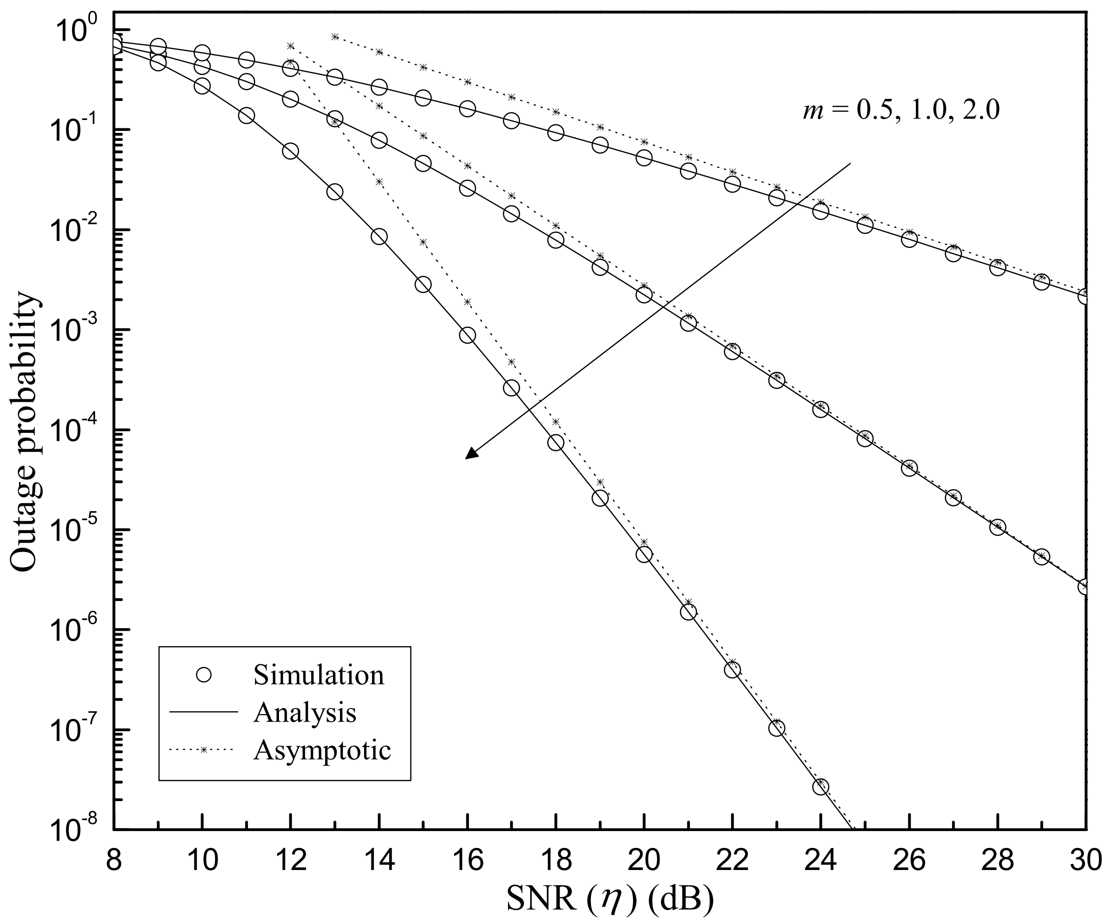
3.4.3 Outage Probability

Fig. 3.8 shows outage probability of a network using the proactive CDF-based relay selection scheme with various fading severity parameters for $K = 3, 5$, respectively. In Fig. 3.8(a) and Fig. 3.8(b), we will use the notation $m_{A,r_k} = m_{B,r_k} = m$ to verify the effect of parameter $m_{i,j}$ on diversity order. Fig. 3.8(a) and Fig. 3.8(b) show that the outage probability analysis of the proactive CDF-based relay selection scheme perfectly matches the simulation results. The asymptotic analysis of the proactive CDF-based relay selection scheme is close to the simulation results at high SNR region. It is shown that as the value of parameter and the number of relays increase, the proactive CDF-based relay selection scheme achieves larger diversity order. Fig. 3.8(c) shows outage probability of a network with various fading severity parameters for $K = 3$. It is shown that the proactive CDF-based relay selection scheme in the case $\mathbf{m}_{A,r_k} = \{0.5, 0.5, 0.5\}$ and $\mathbf{m}_{B,r_k} = \{0.5, 0.5, 0.5\}$ achieves same diversity order as the case $\mathbf{m}_{A,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{B,r_k} = \{0.5, 1, 2\}$, the case $\mathbf{m}_{A,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{B,r_k} = \{2, 1, 0.5\}$, and the case $\mathbf{m}_{A,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{B,r_k} = \{1, 2, 3\}$.

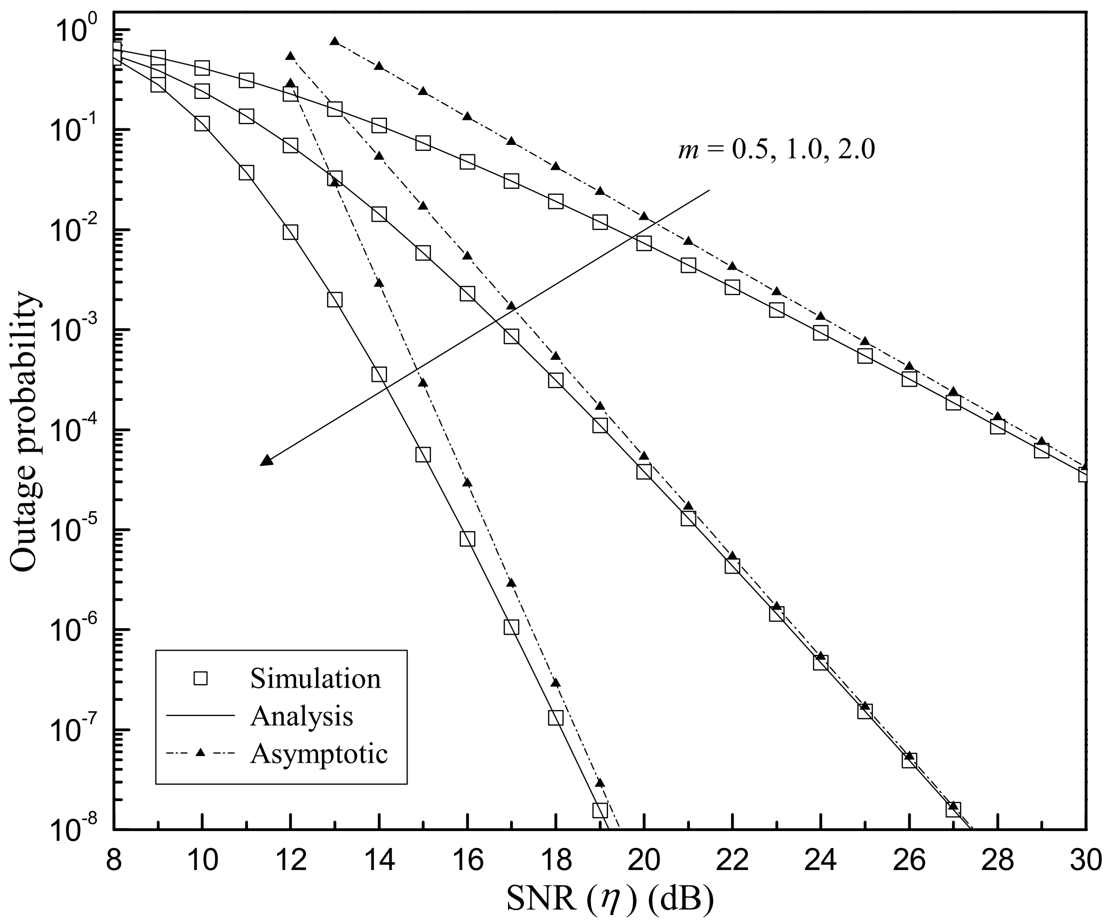
Fig. 3.9 shows outage probabilities of various proactive relay selection schemes. It is shown that proactive max-min SNR relay selection scheme achieves lower outage probability than other relay selection schemes. It is shown that the proactive relay selection schemes for $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{1.0, 2.0, 3.0\}$ achieve lower outage probabilities than those for $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{0.5, 1.0, 1.5\}$.

Fig. 3.10 shows outage probability of a network using the reactive CDF-based relay selection scheme with various fading severity parameters for $K = 3, 5$, respectively.

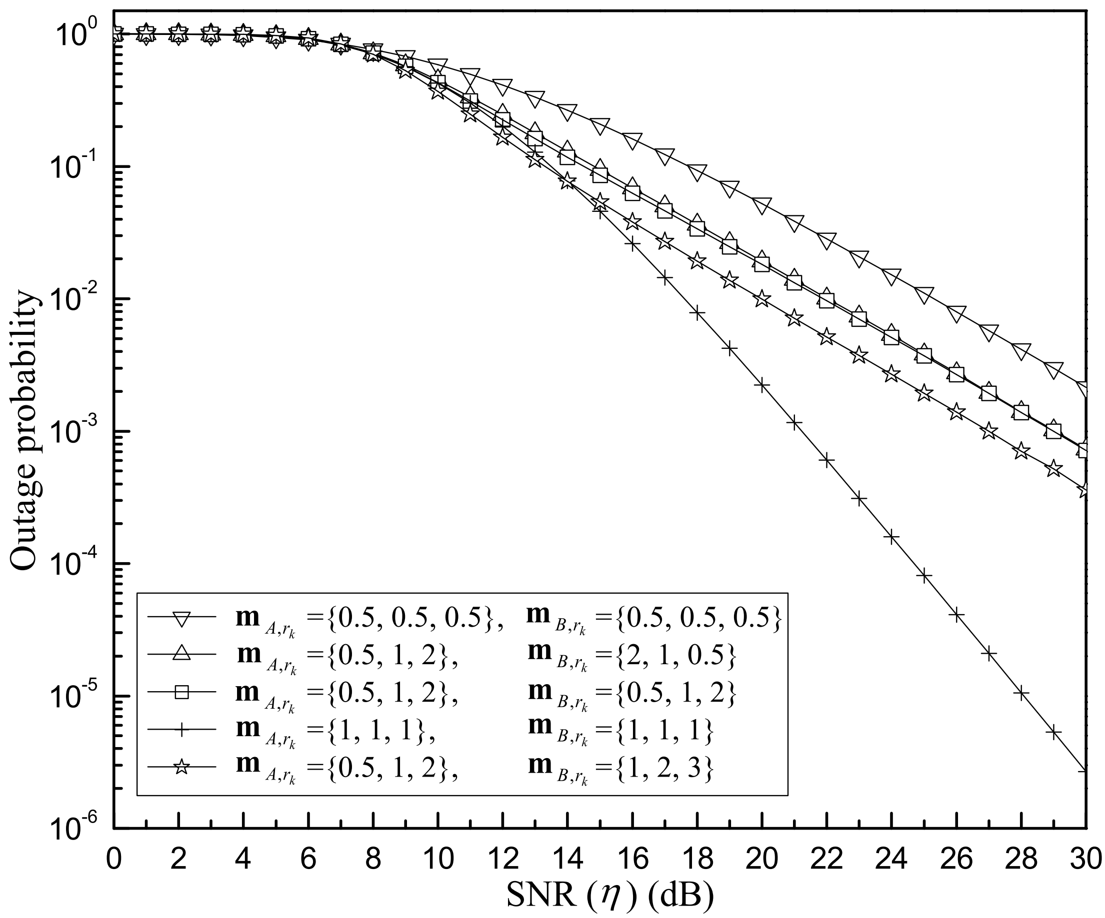
In Fig. 3.10(a) and Fig. 3.10(b), we will use the notation $m_{A,r_k} = m_{B,r_k} = m$ to verify the effect of parameter $m_{i,j}$ on diversity order. Fig. 3.10(a) and Fig. 3.10(b) show that the outage probability analysis of the reactive CDF-based relay selection scheme perfectly matches the simulation results. The asymptotic analysis of the reactive CDF-based relay selection scheme is close to the simulation results at high SNR region. It is shown that as the value of parameter and the number of relays increase, the reactive CDF-based relay selection scheme achieves larger diversity order. Fig. 3.10(c) shows outage probability of a network using the reactive CDF-based relay selection scheme with various fading severity parameters for $K = 3$. It is shown that the reactive CDF-based relay selection scheme in the case $\mathbf{m}_{A,r_k} = \{0.5, 0.5, 0.5\}$ and $\mathbf{m}_{B,r_k} = \{0.5, 0.5, 0.5\}$ achieves same diversity order as the case $\mathbf{m}_{A,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{B,r_k} = \{0.5, 1, 2\}$ and the case $\mathbf{m}_{A,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{B,r_k} = \{2, 1, 0.5\}$. Also, it is shown that the reactive CDF-based relay selection scheme in the case $\mathbf{m}_{A,r_k} = \{1, 1, 1\}$ and $\mathbf{m}_{B,r_k} = \{1, 1, 1\}$ achieves same diversity order as the case $\mathbf{m}_{A,r_k} = \{0.5, 1, 2\}$ and $\mathbf{m}_{B,r_k} = \{1, 2, 3\}$.



(a) $K = 3$, $m_{A,r_k} = m_{B,r_k} = m$

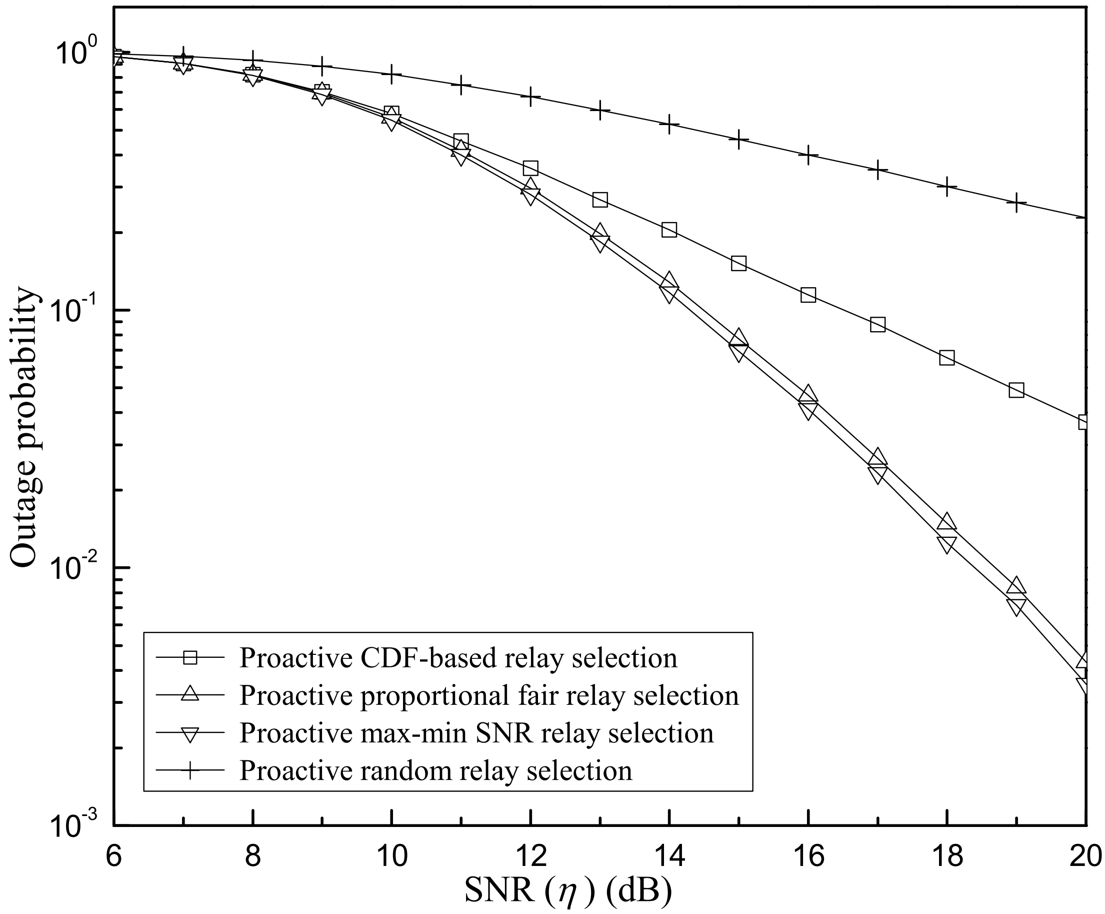


(b) $m_{A,r_k} = m_{B,r_k} = m$, $K = 5$

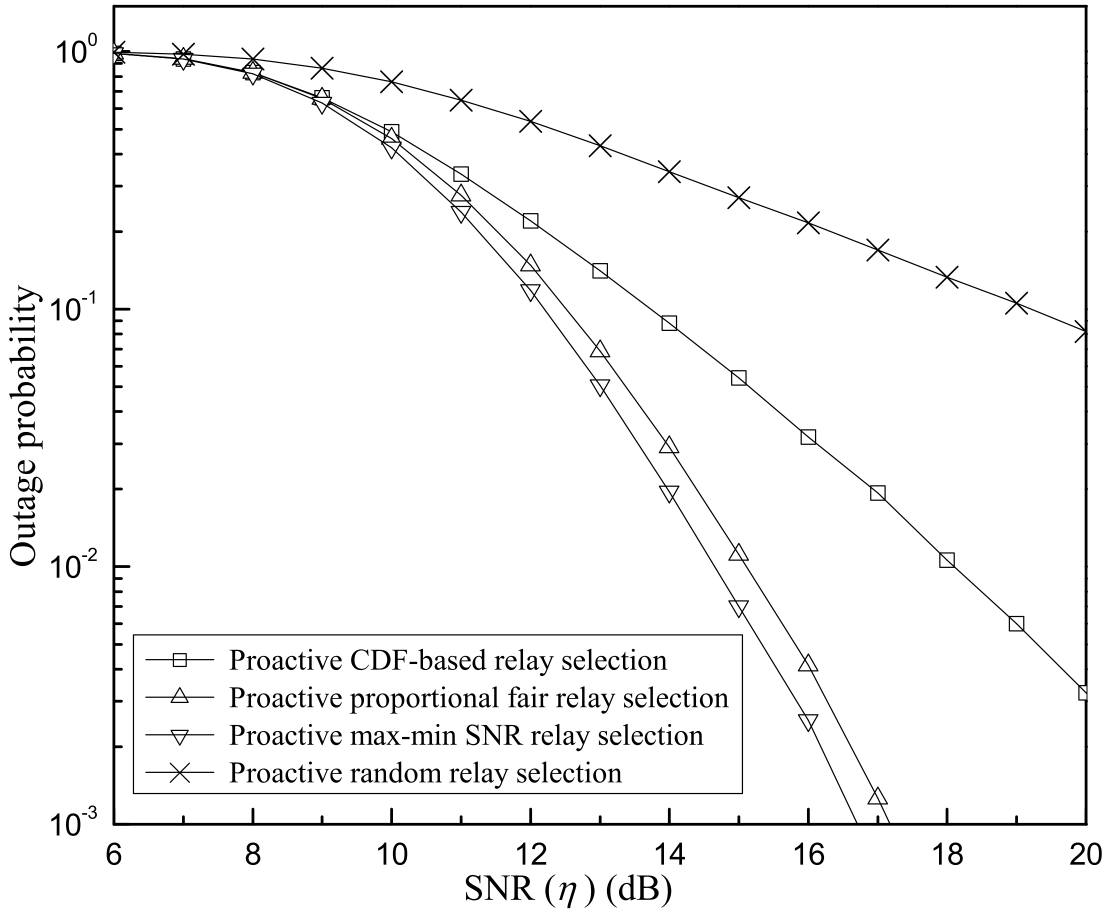


(c) Various values of m_{A,r_k} and m_{B,r_k}, $K = 3$

Figure 3.8. Outage probability of proactive CDF-based relay selection scheme.

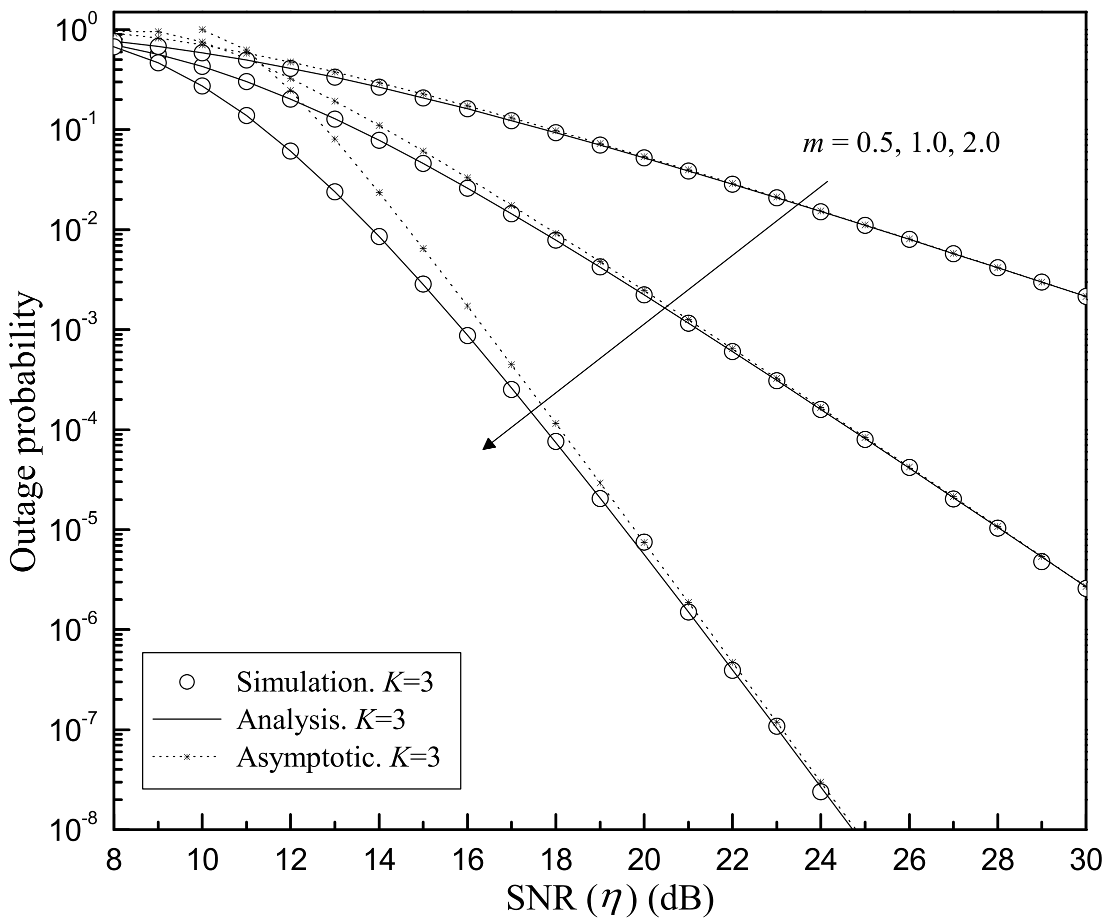


(a) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{0.5, 1.0, 1.5\}$, $K = 3$

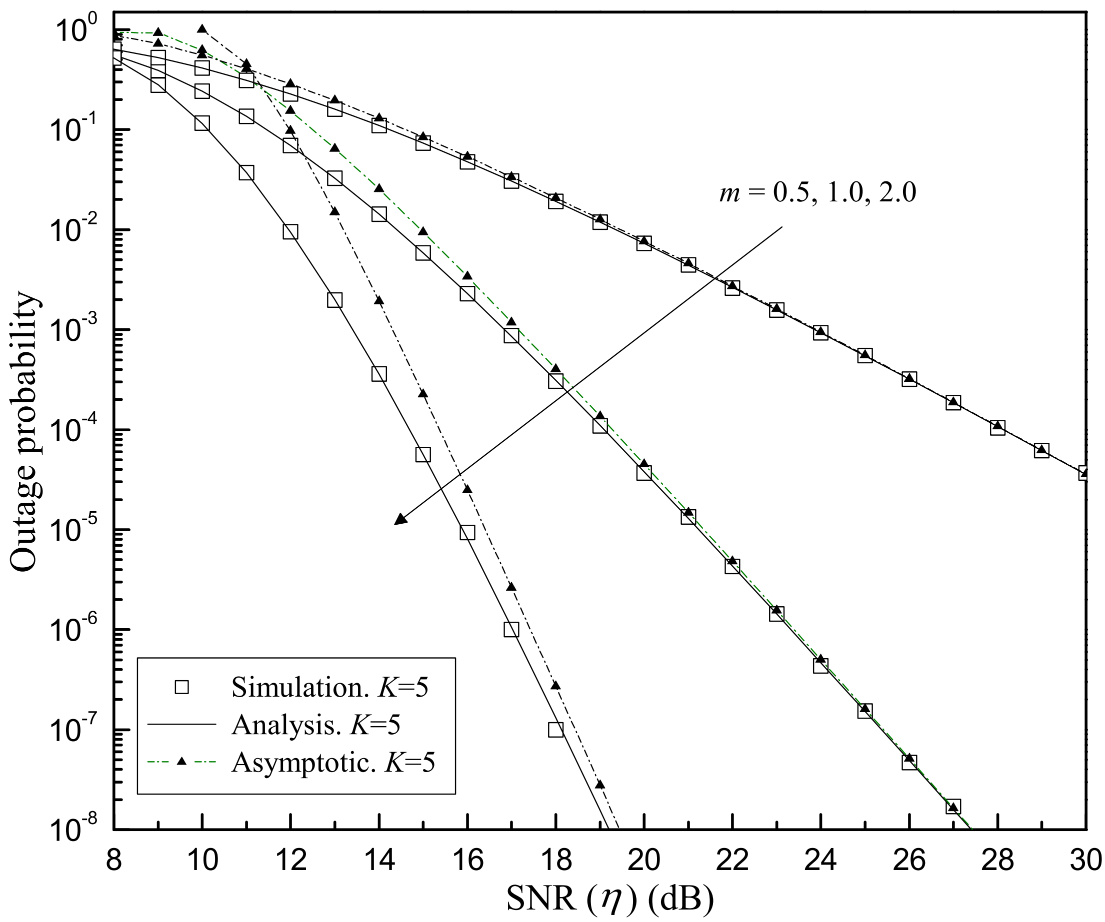


(b) $\mathbf{m}_{A,r_k} = \mathbf{m}_{B,r_k} = \{1.0, 2.0, 3.0\}$, $K = 3$

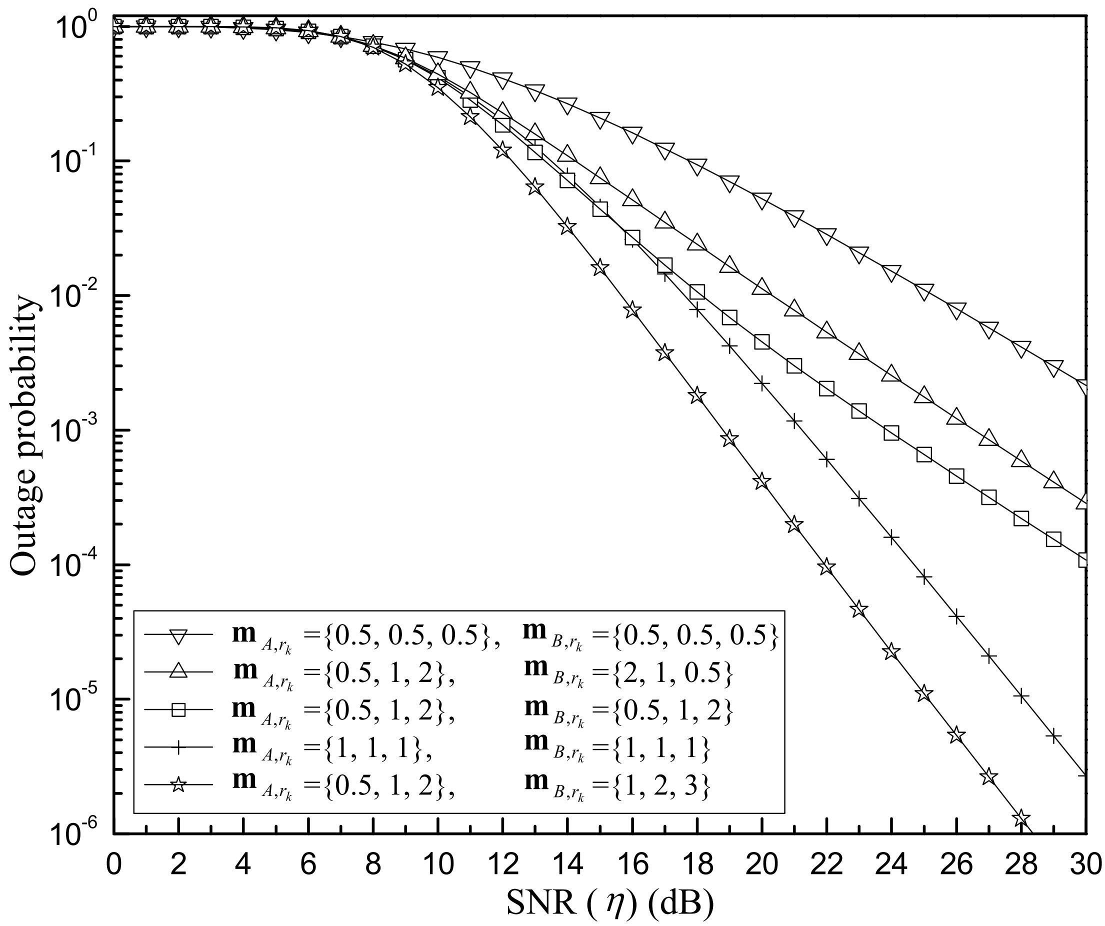
Figure 3.9. Outage probability of various proactive relay selection schemes.



(a) $m_{A,r_k} = m_{B,r_k} = m$, $K = 3$



(b) $m_{A,r_k} = m_{B,r_k} = m$, $K = 5$



(c) Various values of m_{A,r_k} and m_{B,r_k}, $K = 3$

Figure 3.10. Outage probability of reactive CDF-based relay selection scheme.

3.5 Summary

In this chapter, we investigate the proactive and the reactive relay selection schemes based on CDF of SNRs for two-way relay networks over Nakagami-m fading channels. For the proactive CDF-based relay selection scheme, average relay fairness is analyzed by deriving relay selection probability to verify strictness of fairness for potential relays. Also, diversity order is analyzed by deriving the integral and asymptotic expressions for outage probability. For the reactive CDF-based relay selection scheme, average relay fairness analyzed by deriving the exact integral and asymptotic expressions for relay selection probability. Also, diversity order is obtained by deriving the asymptotic expression for outage probability. Analytical results are verified by Monte Carlo simulations. Numerical results show that the analytical results of average relay fairness and outage probability match the simulation results of them well. It is shown that the proactive CDF-based relay selection scheme guarantees strict fairness among relays regardless of the SNR. Whereas, reactive CDF-based relay selection scheme guarantees strict fairness among relay at high SNR region. To provide insights into the impact of the average relay fairness on network lifetime, we show network lifetime performances. It is shown that proactive CDF-based relay selection scheme achieves higher network lifetime than other schemes except proactive random relay selection scheme. Also, it is shown that reactive CDF-based relay selection scheme achieves higher network lifetime than other schemes. With respect to outage probability, it is shown that diversity order depends on the number of relays and fading severity parameters.

Chapter 4

Conclusion

4.1 Summary

In this dissertation, we have investigated the relay technology in the wireless networks.

In Chapter 1, we introduce the basic concept, history, and related works of the wireless relay technology. In addition, we describe the outline of this dissertation and present the notation, abbreviations, and functions used in this dissertation.

In Chapter 2, we propose the proactive and the reactive relay selection schemes based on CDFs of SNRs for one-way relay networks over Nakagami-m fading channels. For both the proactive and the reactive relay selection schemes, average relay fairness is analyzed by deriving relay selection probability. For the proactive CDF-based relay selection scheme, diversity order is analyzed by deriving the integral and asymptotic expressions for outage probability. For the reactive CDF-based relay selection scheme, diversity order is obtained by deriving the exact and asymptotic expressions for outage probability. Analytical results are verified by Monte Carlo simulations. Numerical results show that the analytical results match the simulation results well. It is shown that the proposed schemes guarantee strict fairness among relays and extend network lifetime. Also, it is shown that diversity order depends on the number of relays and fading severity parameters. As the value of parameter and the number of relays increase, the proposed schemes achieve larger diversity order.

In Chapter 3, we propose the proactive and the reactive relay selection schemes based on CDF of SNRs for two-way relay networks over Nakagami-m fading channels. For the proactive CDF-based relay selection scheme, average relay fairness is analyzed by deriving relay selection probability. Also, diversity order is analyzed by deriving the integral and asymptotic expressions for outage probability. For the reactive CDF-based relay selection scheme, average relay fairness analyzed by deriving the exact integral and asymptotic expressions for relay selection probability. Also, diversity order is obtained by deriving the asymptotic expression for outage probability. Analytical results are verified by Monte Carlo simulations. Numerical results show that the analytical results match the simulation results well. It is shown that the proposed schemes guarantee strict fairness among relays and extend network lifetime. Also, it is shown that diversity order depends on the number of relays and fading severity parameters. As the value of parameter and the number of relays increase, the proposed schemes achieve larger diversity order.

4.2 Possible Applications

4.2.1 Device-to-Device (D2D) Communications

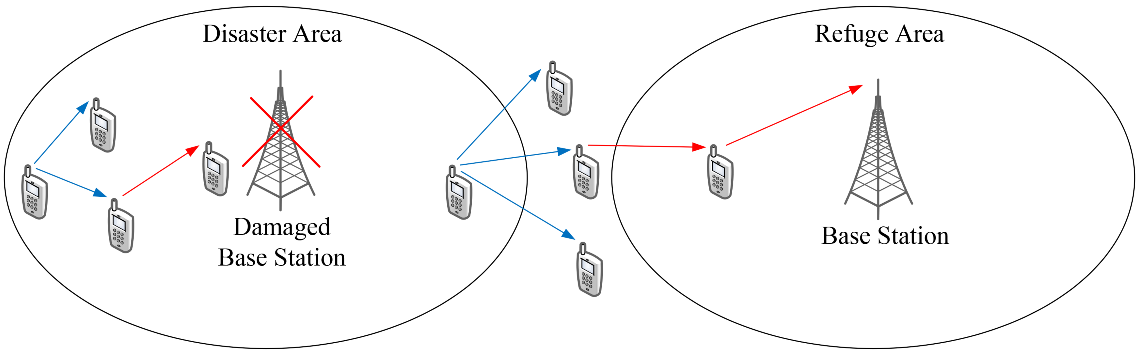
To alleviate the huge infrastructure investment in the exponential growth of mobile traffic and improve local service flexibility, device-to-device (D2D) communications have been considered one of the key techniques in the Third Generation Partnership Project (3GPP) Long Term Evolution Advanced (LTE-Advanced) [110]. In 3GPP Release 12, it has been adopted that D2D communications is of high interest for further investigation [111]. D2D communications allow two nearby mobile devices to communicate with each other in the licensed cellular bandwidth without a BS involved or with limited BS involvement [110]. It make it possible for mobile devices in a network to function as relays for each other.

D2D communications can be adopted to public safety service in disasters [110]. For example, in an earthquake or hurricane, no available infrastructure for communications remains due to physical damage to BSs and insufficient available power. Although disaster victims had devices for communications such as mobile phones, smart phones, and tablets, those devices become practically useless because there is no infrastructure to support communications services. From this, it is needed that the essential functionalities of mobile devices is to be able to transmit a small package of information (text, voice, photo, etc.) toward the outside, even with best effort service and even without acknowledgment [113]. An urgent network can be set up by using

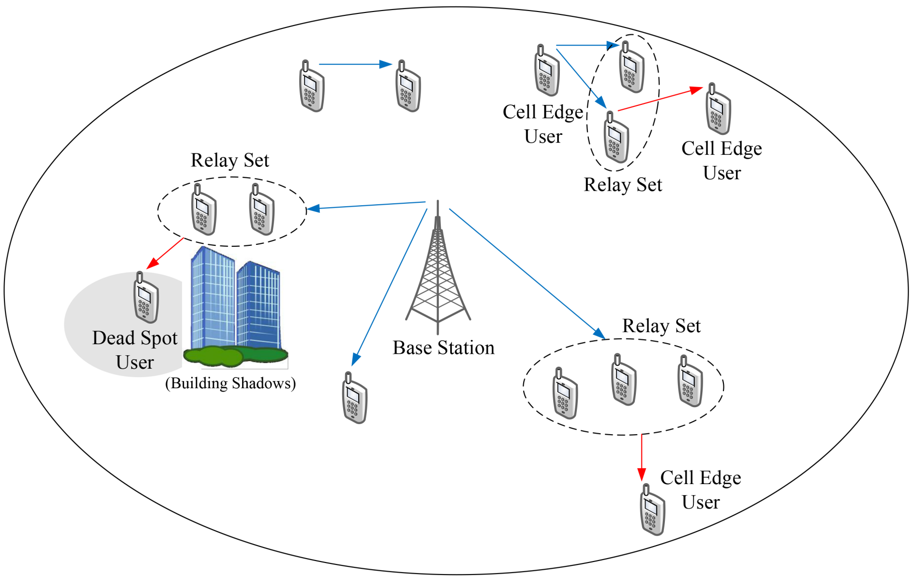
D2D communications in a short time, replacing the damaged networks. Usage methods for relaying are classified into two types: Emergency information transmission and information exchange in a local area. CDF-based relay selection which improves both reliability and network lifetime is suitable for the public safety service.

By using D2D communications, a user at the edge of a cell or in a dead spot can communicate with the BS through relaying its information via other users [112]. One possible incentive for relaying user is that the operator can offer some discounts on monthly bills based on the amount of data they relay through them. Another possible incentive for the relaying users is that instead of a discount on the monthly bill, the operator can offer some free services in exchange for the amount of data they have relayed [112]. CDF-based relay selection is suitable for low-rate data transmission requiring high reliability such as packet-based voice communications at the edge of a cell or a dead spot.

D2D communications can be considered as a cost effective solution for cellular networks offloading where service providers take some load off of the network in a local area such as a stadium or a big mall by allowing direct transmission among mobile devices. D2D communications can play an essential role in mobile cloud computing and facilitate effective sharing of resources (spectrum, power, applications, contents, etc.) for users who are spatially close to each other [112].



(a) Public safety service: Emergency information transmission and information exchange in a local area



(b) Communication at cell edge users (or dead spot users) by using other users

Figure 4.1. Applications of D2D communications.

4.2.2 Low Power Body Sensor Networks

Body sensor networks consist of wireless sensors which are swallowed by patients and send collected data to outside coordinator [114], [115]. The body sensor network is deployed around the human body which exposes communication to severe attenuations due to the shadowing of body parts [116]. This means that transmitting over an arbitrary distance near the human body is not always possible. A major challenge in designing a body sensor network is minimizing the power consumption of the nodes. Relaying is an effective way for reducing transmission power while maintaining high-quality links to arbitrary locations on the body [114]. CDF-based relay selection which reduces transmission power for certain outage probability and improves network lifetime is suitable for body sensor networks.

4.3 Future Work

The enormous potential of relaying for the next-generation wireless systems has been revealed in the recent literature. However, there are still a lot of challenges to be addressed on both the theoretical and practical aspects. The investigations on relay selection based on CDFs of SNRs in wireless relay networks with amplify-and-forward (AF) protocol have not been yet carried out sufficiently. The future work for relay selection based on CDFs of SNRs in wireless relay networks with AF protocol includes: 1) deriving average relay fairness, 2) deriving outage probability, and 3) obtaining diversity order. Also, the investigations on link selection based on CDFs of SNRs in multi-hop relay networks have not been yet carried out sufficiently. The future work for link selection based on CDFs of SNRs in multi-hop relay networks includes: 1) deriving outage probability, 2) analyzing the relation between buffer size and outage probability, and 3) obtaining diversity order.

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Korean Abstract

무선 중계 기술은 차세대 무선통신 시스템에서 요구되는 높은 서비스 품질 및 데이터 전송률 달성을 위해 고려되고 있는 대표적인 기술 중 하나이다. 무선 중계 기술이 갖고 있는 다양한 장점으로 인해 현재까지 IEEE 802.16j 및 3GPP LTE-Advanced 등의 무선통신 시스템 표준에 반영되기도 하였다.

실질적으로 두 노드 사이 채널의 통계적 특성은 그들의 위치에 따라 달라지기 때문에 각 채널들의 통계적 특성은 서로 동일하지 않다. 각 채널들의 통계적 특성이 동일하지 않을 때, 무선 중계 기술에서 가장 유용한 기법 중 하나인 중계기 선택 기법은 특정 중계기들이 더 자주 선택되는 등의 공정성 문제를 유발시킬 수 있다. 특히, 이 문제는 제한된 배터리를 가진 중계기들로 구성된 네트워크에서 네트워크의 수명을 줄이게 하는 요인이 될 수 있다. 따라서 이러한 네트워크에서는 사용자들의 통신 신뢰도 뿐만 아니라, 중계기에서의 선택 공정성도 함께 고려할 필요가 있다.

본 논문에서는 무선 중계 네트워크에서 사용자들의 통신 신뢰도와 중계기 간의 선택 공정성을 함께 고려하기 위해 수신 신호대잡음비의 누적분포함수를 기반으로 하는 새로운 중계기 선택 기법을 제안한다. 주요한 연구 결과는 다음과 같다.

먼저, 나카가미-m 페이딩 채널 환경을 가진 일방향 중계 네트워크를 위한 프로액티브(proactive) 및 리액티브(reactive) 방식의 수신 신호대잡음비 누적분포함수 기반

중계기 선택 기법을 제안한다. 각각의 중계기 선택 기법을 위해 중계기 선택 확률을 유도하여 제안된 각 중계기 선택 기법들의 평균 중계기 공정성을 분석한다. 또한 각 선택 기법에 대한 불능 확률을 수식으로 유도하고, 유도한 불능 확률을 점근적 표현으로 나타내어 각 기법들이 얻을 수 있는 다이버시티 차수를 분석한다. 모의실험을 통해 얻어진 평균 중계기 공정성과 불능 확률이 유도한 평균 중계기 공정성 및 불능 확률 값과 일치함을 확인한다. 그리고 제안된 기법이 중계기들 사이에 공정성을 완벽하게 보장하고 네트워크 수명을 증가시키며, 다이버시티 차수가 중계기의 수와 페이딩 파라미터 m 값에 따라 달라짐을 확인한다.

둘째, 나카가미-m 페이딩 채널 환경을 가진 양방향 중계 네트워크를 위한 프로액티브 및 리액티브 방식의 수신 신호대잡음비 누적분포함수 기반 중계기 선택 기법을 제안한다. 제안된 프로액티브 방식의 중계기 선택 기법에 대해서는 정확한 중계기 선택 확률의 유도를 통해 평균 중계기 공정성을 분석한다. 제안된 리액티브 방식의 중계기 선택 기법에 대해서는 중계기 선택 확률의 적분 및 근사 표현을 유도하여 평균 중계기 공정성을 분석한다. 또한 각 선택 기법에 대한 불능 확률을 수식으로 유도하고, 유도한 불능 확률을 점근적 표현으로 나타내어 각 기법들이 얻을 수 있는 다이버시티 차수를 분석한다. 모의실험을 통해 얻어진 평균 중계기 공정성과 불능 확률이 유도한 평균 중계기 공정성 및 불능 확률 값과 일치함을 확인한다. 그리고 제안된 기법이 중계기들 사이에 공정성을 완벽하게 보장하고 네트워크 수명을 증가시키며, 다이버시티 차수가 중계기의 수와 페이딩 파라미터 m 값에 따라 달라짐을 확인한다.

주요어: 무선 중계 기술, 누적분포함수, 중계기 선택, 일방향 중계, 양방향 중계, 중계기 선택 확률, 평균 중계기 공정성, 불능 확률, 네트워크 수명, 다이버시티 차수.

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